

## Texas Geometry and Topology Conference

This is a report on the presentations at the 53rd meeting of the Texas Geometry and Topology Conference at the University of Houston on February 27-March 1, 2015. This conference was partially supported by National Science Foundation Grant DMS-1203131, and the University of Houston. Speakers reported on recent research. All plenary speakers provided abstracts. Plenary speakers were encouraged to offer in their abstracts slightly broader discussions of the significance and context of their results.

### Meeting 53. University of Houston, February 27-March 1, 2015

#### Bernd Sturmfels, UC Berkeley, *Tropical Plane Curves - Introductory talk*

Tropical geometry is a combinatorial shadow of classical geometry. Algebraic curves in the tropical plane are dual to triangulations of convex polygons. We discuss the intrinsic geometry of these objects, with focus on the moduli space of metric graphs that represent tropical plane curves.

#### Bernd Sturmfels, UC Berkeley, *The 27 trees on a cubic surface - Advanced talk*

We explore the tropicalization of moduli spaces in classical algebraic geometry. Our primary example is the moduli space of marked del Pezzo surfaces of degree 3. Its universal family tropicalizes to a 6-dimensional fan with millions of cones. Its fibers are the tropical cubic surfaces. These are characterized by their arrangement of 27 trees, one for each  $(-1)$  curve. This is joint work with Qingchun Ren, Steven Sam and Kristin Shaw.

#### Laura Matusevich (Texas A&M), *A-hypergeometric solution sheaves*

I will discuss recent work that aims to explain the behavior of the solutions of an A-hypergeometric system as functions of the parameters. Our goal, which can be achieved in special cases, is to stratify the parameter space in such a way that these solutions are locally analytic within each (connected component of a) stratum. Moreover, we wish to understand the transitions between strata, and also have control over what happens at the intersections of connected components within a stratum. A major obstacle is that the dimension of the solution space of such systems is not constant as the parameters vary. This is part of a joint project with Christine Berkesch Zamaere and Jens Forsgard.

#### Steve Zelditch (Northwestern), *Plank's constant, time, and stationary states in quantum mechanics*

Quantum mechanics solved the problem of how a particle can move and be stationary at the same time. It did this by replacing geometry (classical mechanics) by linear algebra (eigenfunctions, eigenvalues and spectral theory). But intuition asks for a geometric picture of the time evolution of quantum states and the topography of eigenfunctions. As Planck's constant  $\hbar \rightarrow 0$ , quantum mechanical objects have asymptotic relations to classical mechanical objects and provide the best picture possible.

My talk will concern the topography of eigenfunctions of the Laplacian on Riemannian manifolds  $(M, g)$ : their sizes and shapes as measured by their zero sets, sup norms,  $L^p$  norms. One theme is to describe the  $(M, g)$  possessing extremal eigenfunctions. Another is the real and complex geometry of zero sets. The methods come from micro local analysis and complex geometry.

**Colleen Robles (Texas A&M), *Characteristic cohomology of the horizontal subbundle on flag manifolds and connections to Hodge theory***

A (complex) flag manifold is a homogeneous manifold  $G/P$  with  $G$  a complex semisimple Lie group. (For example, the grassmannian  $Gr(k, n)$  of  $k$ -planes in  $n$ -space is homogeneous under the action of  $G = SL(n, C)$ .) The horizontal subbundle  $H$  is the unique, minimal, homogeneous, bracket-generating subbundle of the tangent bundle  $T(G/P)$ .

The horizontal subbundle determines a quotient of the de Rham complex on  $X$ , and the characteristic cohomology (CC) is the cohomology of this quotient complex. One can think of the CC as the cohomology that induces ordinary cohomology on an integral  $X$  of  $H$  by virtue of  $X$  being a solution to a system of PDE.

The CC can be realized as the cohomology of a complex of differential operators. (This complex is related to the Rumin complex and its cousins.) Basic questions for such a complex are: When is the cohomology finite dimensional? When does it vanish? When does a local Poincare lemma hold? I will discuss these questions and, if time allows, a connection with Hodge theory.

**Mark Stern (Duke University), *Introduction to nonlinear harmonic forms***

We motivate and introduce nonlinear harmonic forms. These are de Rham representatives  $z$  of cohomology classes which minimize the energy  $\|z\|_{L_2}^2$  subject to a nonlinear constraint. We give basic existence results for quadratic constraints, discuss the rich Euler Lagrange equations, and ask many regularity questions.

**Cameron Gordon (University of Texas, Austin), *Left-orderability and 3-manifold groups***

The fundamental group is a more or less complete invariant of a 3-dimensional manifold. We will discuss how the purely algebraic property of this group being left-orderable is related to two other aspects of 3-dimensional topology, one geometric-topological and the other essentially analytic. More precisely, it is conceivable that for a rational homology 3-sphere  $M$  the following are equivalent: (1)  $\pi_1(M)$  is left-orderable, (2)  $M$  supports a co-orientable taut foliation, and (3)  $M$  is not a Heegaard Floer homology  $L$ -space. We will describe some results concerning these possible equivalences.

**Lei Ni (UCSD), *Hypersurface flow by the power of the Gauss curvature.***

The Gauss curvature flow was initiated by Firey to model the tumbling of stones. For the flow by the special powers of the Gauss curvature it was studied by Ben Chow for the power of  $1/n$ , and by Ben Andrews for the affine normal (power of  $\frac{1}{n+2}$ ) as well as other small powers. In this talk I shall explain a unified method which proves the convergence for general powers in any dimension. This is a joint work with Ben Andrews and Pengfei Guan.

**Alastair Hamilton (Texas Tech University) *Functional integrals and pairing classes in the moduli space of Riemann surfaces.***

Using an algebraic model for the homology of compactified moduli spaces of Riemann surfaces that originated in Kontsevich's papers on formal noncommutative symplectic geometry, we will describe two constructions producing homology and cohomology classes in these moduli spaces. These constructions were originally considered by Kontsevich for the open moduli space, and we will describe versions of them here for the compactified spaces by making use of the so-called "Batalin-Vilkovisky formalism", which has its origins in the quantization of gauge systems. The pairing of these two constructions may be computed by a

certain (well-defined) functional integral, and we will present the first examples of such computations since the idea was first proposed by Kontsevich.