Texas Geometry and Topology Conference

This is a report on the presentations at the 57th meeting of the Texas Geometry and Topology Conference at Texas Tech University on February 17-19, 2017. This conference was partially supported by National Science Foundation Grant DMS-1510060, and Texas A&M University. Speakers reported on recent research. All plenary speakers provided abstracts. Plenary speakers were encouraged to offer in their abstracts slightly broader discussions of the significance and context of their results.

Meeting 57. Texas Tech University, February 17-19, 2017

Josef Dorfmeister, Duke University, *Harmonic Maps from Riemann Surfaces to k-Symmetric Spaces via Loop Groups Values*

Harmonic maps between Riemannian manifolds have been investigated intensively for several decades now. A particularly interesting application of harmonic map theory is to surface theory. The classical result, due to Ruh, states that an immersion $f : M \to R^3$ from a Riemann surface M to R^3 has constant mean curvature if and only if the Gauss map $N : M \to S^2$ of f is harmonic (relative to the naturally given metrics). It turns out that many surface classes admit a similar characterization, and we will present several examples. An important feature of such surfaces is that they are a member of an S^1-*family* of harmonic maps. And then one obtains an S^1-*family* of framings of the harmonic maps. The fact that the domain is a surface permits a very useful description of the Maurer-Cartan forms of these frames. Thus the discussion centers on a description/construction of all S^1-*families* of frames of harmonic maps and is achieved by considering these families as maps into loop groups. As an illustration of these results we will present some concrete applications to surface theory.

David Gabai, Princeton University, *Maximal Cusps of Low Volume*

This work is joint with Robert Haraway, Robert Meyerhoff, Nathaniel Thurston and Andrew Yarmola. We address the following question. What are all the 1-cusped hyperbolic 3-manifolds whose maximal cusps have low volume? Among other things we will outline a proof that the figure-8 knot complement and its sister are the 1-cusped manifolds with minimal maximal cusp volume.

Ryan Grady, Montana State University, *Topological invariants via local-to-global constructions of QFT*

The relationship between certain topological invariants and quantum field theory (QFT) is one that goes back more than 30 years. Particularly relevant are a certain class of theories called sigma models, whose fields are spaces of maps between manifolds. In this talk we will describe several such models and their quantizations in the BV formalism. Moreover, we will describe how these QFTs encode the topology/geometry of the target manifold. A key tool will be a localization technique which simplifes the construction and quantization of these types of QFT.

Eleny Ionel, Stanford University, *The Gopakumar-Vafa Conjecture for Symplectic Manifolds*

In the late nineties physicists Gopakumar and Vafa conjectured that the Gromov-Witten invariants of Calabi-Yau 3-folds have a hidden structure: they are obtained, by a specific transform, from a set of more fundamental “BPS numbers”, which are integers. In joint work with Tom Parker, we proved this conjecture by decomposing the GW invariants into contributions of “clusters” of curves, deforming the almost complex structure and reducing it to a local calculation. This talk presents some of the background and geometric ingredients of our proof, as well as recent progress, joint with Penka Georgieva, towards proving that a similar structure theorem holds for the real GW invariants of Calabi-Yau 3-folds with an anti-symplectic involution.

Irena Kogan, Duke University, *Jacobians with Prescribed Eigenvectors*

We consider the problem of constructing all possible maps from an open subset $\Omega \subset \mathbb{R}^n$ to $\mathbb{R}^n$, such that the set of eigenvector fields of the Jacobian matrix of each of these maps contains a given set of $m \leq n$ independent vector fields on Ω. Our initial motivation for considering this problem comes from the geometric study of hyperbolic conservative systems $u_t + f(u)_x = 0$ in one spatial dimension. This problem is, however, of independent geometric interest and, in turn, leads to interesting overdetermined systems of PDEs, which can be studied via classical integrability theorems, such as Frobenius and Darboux theorems, and their appropriate generalizations. This is joint work with Michael Benfield and Kris Jenssen.

Jeff Lagarias, University of Michigan, *Configuration Spaces and Materials Science*

Configuration spaces of points on a manifold were studied in topology beginning in the 1960's. They appear in many fields, including algebraic geometry, graph theory and robotics, and here materials science. This talk discusses constrained configuration spaces of N spheres of equal radius r touching a central sphere of radius 1. These topological spaces form a family with r as the constraint parameter, and with $r = 0$ being the classical configuration space. They can be viewed as toy models for studying some aspects of granular materials, having a packing density of spherical caps determined by N and r (the Tammes problem), with N acting as a discretization parameter. The special case $N = 12$ and radius $r = 1$ has a long history in connection with the three-dimensional sphere packing problem and is of special interest. The talk discusses the topology and geometry of these spaces for small N as the parameter r varies, with particular emphasis on the case $N = 12$ with $r = 1$. (This is joint work with Rob Kusner, Woden Kusner and Senya Shlosman.)

William Meeks, University of Massachusets at Amerst, *Recent Progress in the Theory of Constant Mean Curvature Surfaces*

In this talk I will discuss joint work with Joaquin Perez, Antonio Ros, Giuseppe Tinaglia and Pablo Mira. Joint work with Perez and Ros (and Harold Rosenberg) has led to the completing of the classification of properly embedded minimal surfaces of genus 0; these examples are planes, catenoids, helicoids, and Riemann minimal examples. Joint work with Tinaglia proves that compact disks of constant mean curvature 1 embedded in $\mathbb{R}^3$ have curvature estimates away from their boundaries and that there exists a universal bound on the intrinsic radius of such disks. Consequently, any complete, simply-connected embedded surface in $\mathbb{R}^3$ of positive constant mean curvature have bounded second fundamental forms if and only if they have positive injectivity radius. We also prove that a complete embedded surface in $\mathbb{R}^3$ is proper if it has finite topology or positive injectivity radius. Joint work with Perez and Ros gives the existence of removable understanding of the local structure of CMC foliations of 3-manifolds near any isolated singularity. My talk ends with an outline of my recent proof with Mira, Perez and Ros of the Hopf Uniqueness Theorem in homogenous 3-manifolds. This generalization proves that two such spheres of the same mean curvature are congruent and provides a description of the associated 1-dimensional moduli spaces.

Peter Petersen, UCLA, *The Alekxeevsky Conjecture*

It has been a long standing goal to understand which homogeneous spaces admit Einstein metrics. In the cases where the Einstein constant is nonnegative this has been understood for quite a while. In the negative case much less progress has been made. Alekseevsky conjectured upwards of 45 years ago what such spaces should look like. The talk will explain all of this and also what progress has been made on this conjecture in recent years.