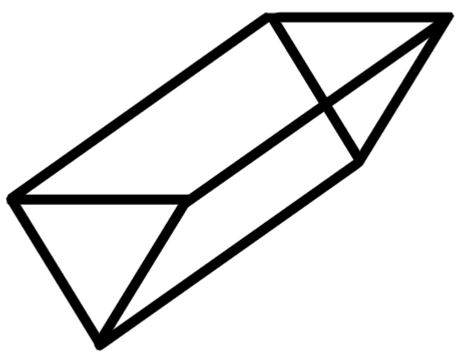
MATH 151, SPRING 2021
COMMON EXAM 2 - ONLINE EXAM VERSION A

The work out problems make up 44 points of the exam, while the multiple choice problems make up 56 points (3.5 points each) for a total of 100 points. **No calculator is allowed!**

PART I: WORK OUT PROBLEMS

Directions: Present each of your solutions on an empty sheet/side of paper. *Show all of your work* neatly and concisely and *box your final answer.* You will be graded not merely on the final answer, but also the quality and correctness of the work leading up to it.

1. (8 pts) Find the values of a and b so that the line $5x + 2y = a$ is tangent to the function $y = bx\sqrt{x}$ when $x = 9$.
2. (8 pts) Find $\dfrac{dy}{dx}$ for the equation: $\tan(7x^2) = 3^{2y} + y^5 e^{6x}$.
3. (9 pts) A trough is 18 m long and its ends have the shape of isosceles triangles that are 8 m across the top and have a height of 3 m. Water is being drained from it at a rate of 12 m^3/min. Find the rate at which the height of the water in the tank is changing when the height of the water is 1 m.



For the remaining work out problems, find the derivative, but do not simplify during or after taking the derivative. Your final answer should also not include y and should only be in terms of x.

4. (6 pts) Find $\dfrac{dy}{dx}$ for $y = \dfrac{\ln(\pi) - x^7}{\sec(2x)\ln(3x)}$.
5. (5 pts) Find $\dfrac{dy}{dx}$ for $y = \arctan(x^2 e^{4x})$.
6. (8 pts) Find $\dfrac{dy}{dx}$ for $y = (4 - 3x)^{\sin(5x)}$.

PART II: Multiple Choice. 3.5 points each

Use the table of values given below for differentiable functions f and g to answer Questions #7 and #8.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-2	-6	12	3	-3
1	1	9	2	6
2	-3	2	1	-2

7. Let $u(x) = f(g(2x))$. Find $u'(1)$.

   (a) -4
   (b) -36
   (c) 12
   (d) -18
   (e) -8

8. Let $v(x) = \dfrac{f(x)}{g(x)}$. Find $v'(-2)$.

   (a) 2
   (b) 6
   (c) 5
   (d) -6
   (e) -2

9. Find the t-value(s) so that the curve $x = 2t^3 + 6t^2$, $y = t^3 - 2t$ has a vertical tangent.

   (a) $t = -3, 0$
   (b) $t = -1, 0, 1$
   (c) $t = -1, 1$
   (d) $t = -\sqrt{3}, 0, \sqrt{3}$
   (e) $t = -2, 0$

10. The length of a rectangle is increasing at a rate of 7 cm/s and its width is decreasing at a rate of 3 cm/s. When the length is 12 cm and the width is 5 cm, at what rate is the area of the rectangle changing at that moment?

    (a) -69 cm/s
    (b) 71 cm/s
    (c) 1 cm/s
    (d) -1 cm/s
    (e) 69 cm/s

11. For what values of x on the interval $[0, 2\pi)$ does the graph of $f(x) = 2\cos(x) + x$ have a horizontal tangent?

(a) $\frac{\pi}{2}, \frac{3\pi}{2}$
(b) $\frac{4\pi}{3}, \frac{5\pi}{3}$
(c) $\frac{\pi}{3}, \frac{2\pi}{3}$
(d) $\frac{7\pi}{6}, \frac{11\pi}{6}$
(e) $\frac{\pi}{6}, \frac{5\pi}{6}$

12. Find the equation of the tangent line to the graph of $y^2\cos(x) = 4x + y$ at the point $(0, 1)$.

(a) $y = -3x + 1$
(b) $y = 4x - 4$
(c) $y = 4x + 1$
(d) $y = 4x$
(e) $y = -3x$

13. Find $f^{(1034)}(x)$, the 1034th derivative of $f(x) = xe^{-x}$.

(a) $f'(x) = (x - 1034)e^{-x}$
(b) $f'(x) = 1034x^{-x}$
(c) $f'(x) = (1034 + x)e^{-x}$
(d) $f'(x) = (1034 - x)e^{-x}$
(e) $f'(x) = -1034x^{-x}$

14. The position of a particle is given by the vector function $\mathbf{r}(t) = \langle t^4, te^t \rangle$. Find the acceleration vector of the particle at time $t = 1$.

(a) $\mathbf{a}(t) = \langle 3, 2e \rangle$
(b) $\mathbf{a}(t) = \langle 12, 2e \rangle$
(c) $\mathbf{a}(t) = \langle 4, 3e \rangle$
(d) $\mathbf{a}(t) = \langle 4, 2e \rangle$
(e) $\mathbf{a}(t) = \langle 12, 3e \rangle$

15. At 1:00 PM, a bacteria culture contains 2 million cells and grows at a rate proportional to its size. At 4:00 PM, it has grown to 5 million cells. How many cells will there be in the culture at 7:00 PM, assuming the same rate of growth? All times are on the same day.

(a) 7 million
(b) 8.5 million
(c) 10 million
(d) 12.5 million
(e) 15 million

16. Find $f''(x)$ for $f(x) = e^{-5\cos(x)}$.

(a) $5\cos(x)e^{-5\cos(x)} + 25\sin^2(x)e^{-5\cos(x)}$
(b) $5\sin(x)e^{-5\cos(x)}$
(c) $\cos(x)e^{-5\cos(x)} + \sin^2(x)e^{-5\cos(x)}$
(d) $e^{5\cos(x)}$
(e) $25\sin^2(x)e^{-5\cos(x)}$

17. Find the tangent vector of unit length for $\mathbf{r}(t) = \langle t+2, e^{3t} \rangle$ at $t = 0$.

(a) $\left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$
(b) $\left\langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \right\rangle$
(c) $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$
(d) $\langle 1, 3 \rangle$
(e) $\langle 2, 1 \rangle$

18. Find the derivative of $f(x) = \ln\left(\frac{\sqrt{x^4 - 6x}}{e^{2x}(x-5)^3}\right)$.

(a) $f'(x) = \frac{2x^3 - 3}{x^4 - 6x} + \frac{3}{x-5} - 2$
(b) $f'(x) = \frac{4x^3 - 6}{x^4 - 6x} - \frac{1}{x-5} - e^{-2x}$
(c) $f'(x) = \frac{1}{2(x^4 - 6x)} - \frac{3}{x-5} - 2e^{-2x}$
(d) $f'(x) = \frac{2x^3 - 3}{x^4 - 6x} - \frac{3}{x-5} - 2$
(e) $f'(x) = \frac{4x^3 - 6}{x^4 - 6x} - \frac{3}{x-5} - 2x$

19. Determine where the function f **IS** differentiable.

$$f(x) = \begin{cases} 6 - 3x, & x < 0 \\ 2x^2 - 3x + 6, & 0 \le x < 1 \\ 7x - 2x^3, & 1 \le x \le 2 \\ x + x^3, & x > 2 \end{cases}$$

(a) $x = 0$ and $x = 1$ only

(b) $x = 0$ and $x = 2$ only

(c) $x = 1$ and $x = 2$ only

(d) $x = 1$ only

(e) $x = 0$ only

20. Find the derivative of $f(x) = \arccos(x^3)$.

(a) $\dfrac{-3x^2}{\sqrt{1-x^2}}$

(b) $\dfrac{3x^2}{\sqrt{1-x^2}}$

(c) $\dfrac{-3x^2}{\sqrt{1-x^6}}$

(d) $\dfrac{3x^2}{\sqrt{1-x^6}}$

(e) $\dfrac{-1}{\sqrt{1-x^6}}$

21. Find the slope of the tangent line to $f(x) = e^{x^2}\ln(x^3+2)$ at $x = -1$.

(a) $e\ln(3) + e$

(b) $3e$

(c) $-2e\ln(3) + 3e$

(d) $-2e\ln(3)$

(e) $-6e$

22. Find the speed of the particle at time t with position function $\mathbf{r}(t) = \langle 5\sin(t), -3\cos(t)\rangle$.

(a) $\sqrt{25\sin^2(t) + 9\cos^2(t)}$

(b) $\langle -5\cos(t), -3\sin(t)\rangle$

(c) $\sqrt{25\cos^2(t) + 9\sin^2(t)}$

(d) $\langle 5\cos(t), 3\sin(t)\rangle$

(e) $\sqrt{34}$