MATH 151, SPRING 2021
COMMON EXAM 3 - ONLINE EXAM VERSION A

The work out problems make up 36 points of the exam, while the multiple choice problems make up 64 points (4 points each) for a total of 100 points. **No calculator is allowed!**

PART I: WORK OUT PROBLEMS

Directions: Present each of your solutions on an empty sheet/side of paper. *Show all of your work* neatly and concisely and *box your final answer*. You will be graded not merely on the final answer, but also the quality and correctness of the work leading up to it.

1. (10 pts) A rectangular box with an open top is to have a volume of 90 cubic meters. The length of the box is triple its height. Material for the base costs 2 dollars per square meter, and the material for the sides cost 3 dollars per square meter. Determine the height of the box that minimizes the cost of the container. Justify that your answer gives a minimum using calculus.

2. (8 pts) Calculate $\lim_{x\to 0^+} [1+f(x)]^{-5/x}$ if $f(x)$ has a continuous first derivative and satisfies $f(0)=0$ and $f'(0)=2$.

3. (9 pts) A particle is moving at a speed of 32 meters per second before it begins slowing down. The particle decelerates at a non-constant rate of $\boldsymbol{12t^2}$ meters per second squared, where t is the time from when it begins slowing down. How far does the particle travel before coming to a stop?

4. (9 pts) Consider a function $f(x)$ that has a vertical asymptote at $x=3$, and for $x \neq 3$ has derivatives:

$$f'(x) = \frac{(x+11)(x-1)^2}{(x+3)^3} \qquad \text{and} \qquad f''(x) = \frac{96(x-1)}{(x+3)^4}.$$

For each of the following, make sure to justify your answers.

(a) Determine the interval(s) where f is increasing and decreasing.

(b) Determine the x-values(s) where any local extrema occur, or argue that there are none. Also state which type of extrema occurs.

(c) Determine the interval(s) where f is concave up and concave down.

(d) Determine the x-coordinate(s) of any inflection points, or argue that there are none.

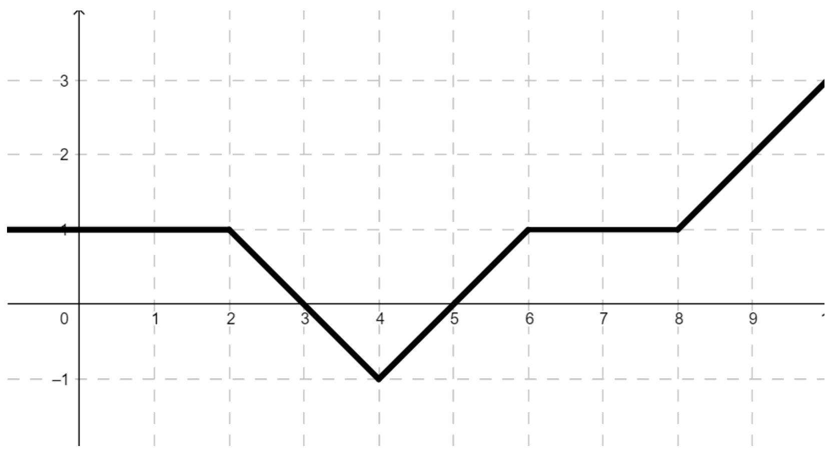
PART II: Multiple Choice. 4 points each

5. Find the most general antiderivative of the function $f(x) = \dfrac{8x - 2x^3\sec(x)\tan(x) + 6x^2}{x^3}$.

   (a) $F(x) = -\dfrac{8}{x} - 2\sec(x) + 2x^3 + C$
   (b) $F(x) = \dfrac{4x^2 - \frac{1}{2}x^4\sec(x) + 2x^3}{\frac{1}{4}x^4}$
   (c) $F(x) = \dfrac{8}{x} - 2\tan(x) + 6\ln|x| + C$
   (d) $F(x) = -\dfrac{8}{x} - 2\sec(x) + 6\ln|x| + C$
   (e) $F(x) = -\dfrac{8}{x} - 2\tan(x) + 2x^3 + C$

6. Use the graph of $f(x)$ below to evaluate $\displaystyle\int_1^8 f(x)\,dx$.

   (a) 3
   (b) 7
   (c) 4
   (d) 5
   (e) $\dfrac{9}{2}$



7. If f has domain all reals **except** $x = 3$ and $f'(x) = -\dfrac{x}{(x-3)^2}$ when $x \neq 3$, find the interval(s) where f is concave up.

   (a) $(-\infty, -3), (0, \infty)$
   (b) $(-\infty, -3), (3, \infty)$
   (c) $(3, \infty)$
   (d) $(0, 3)$
   (e) $(-3, 3)$

8. Evaluate $\lim\limits_{t\to\infty} (t \cdot g(t))$ if $g(t)$ is continuously differentiable with $\lim\limits_{t\to\infty} g(t) = 0$ and $g'(t) = 2(1+t^2)^{-1}$.

   (a) -2
   (b) 0
   (c) ∞
   (d) 2
   (e) $-\infty$

9. The domain of $f(x)$ is all real numbers and $f''(x) = -x(x+4)(x^2-16)$. Find the x-coordinate(s) of all inflection points for the function $f(x)$.

   (a) $x = -4$ and $x = 4$
   (b) $x = -4$, $x = 0$, and $x = 4$
   (c) $x = -4$, $x = 0$, and $x = 16$
   (d) $x = 0$ and $x = 4$
   (e) $x = -4$ and $x = 0$

10. Approximate the area under the curve $f(x) = x^3 + 30$ on the interval $[-3, 3]$ using three rectangles of equal width and left endpoints.

   (a) 63
   (b) 234
   (c) 180
   (d) 240
   (e) 126

11. The domain of $f(x)$ is $(2, \infty)$ and $f'(x) = \dfrac{(x-7)(3-x)}{x-2}$. Find the x-value(s) where the function f has as a local maximum.

   (a) $x = 2$ only
   (b) $x = 3$ and $x = 7$
   (c) $x = 3$ only
   (d) $x = 7$ only
   (e) $x = 2$ and $x = 7$

12. Evaluate $\lim\limits_{x \to 0} \dfrac{e^{-x} + \sin(x) - 1}{3 - 4x^2 - 3\cos(x)}$.

   (a) $-\dfrac{1}{5}$
   (b) $-\infty$
   (c) $-\dfrac{1}{3}$
   (d) $-\dfrac{1}{4}$
   (e) 0

13. Find the absolute minimum and maximum values of the function $f(x) = 2x^3 + 6x^2 - 18x$ on the interval $[-1, 2]$.

(a) 0, 22
(b) -10, 0
(c) -10, 54
(d) -10, 22
(e) $4, 22$

14. The acceleration of a particle is given by $a(t) = 12t^2 - 7\sin(t)$ with $v(0) = -3$ and $s(0) = 5$. Find the position function for the particle.

(a) $s(t) = t^4 + 7\sin(t) - 3t + 5$
(b) $s(t) = t^4 - 7\sin(t) - 10t + 5$
(c) $s(t) = t^4 + 7\sin(t) - 17t + 5$
(d) $s(t) = t^4 - 7\sin(t) - 3t + 5$
(e) $s(t) = t^4 + 7\sin(t) - 10t + 5$

15. Find the interval(s) where the function $f(x) = e^x(x^2 - 7x + 8)$ is concave downwards.

(a) $(0, 4)$
(b) $(-1, 4)$
(c) $(-\infty, -1), (0, \infty)$
(d) $(-\infty, -1)$
(e) $(-\infty, -1), (4, \infty)$

16. Which of the following gives the exact net area under the curve $f(x) = \cos(x)$ on the interval $[1, 6]$?

(a) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{6}{n} \cos\left(1 + \frac{6i}{n}\right)$
(b) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{5}{n} \cos\left(1 + \frac{5i}{n}\right)$
(c) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{6}{n} \cos\left(\frac{6i}{n}\right)$
(d) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{5}{n} \cos\left(\frac{5i}{n}\right)$
(e) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{6}{n} \cos\left(1 + \frac{5i}{n}\right)$

17. Find a number c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \ln(x) - x$ on the interval $[1, e]$.

(a) $c = 0$
(b) $c = e$
(c) $c = e - 1$
(d) $c = \dfrac{1}{e-1}$
(e) $c = 1$

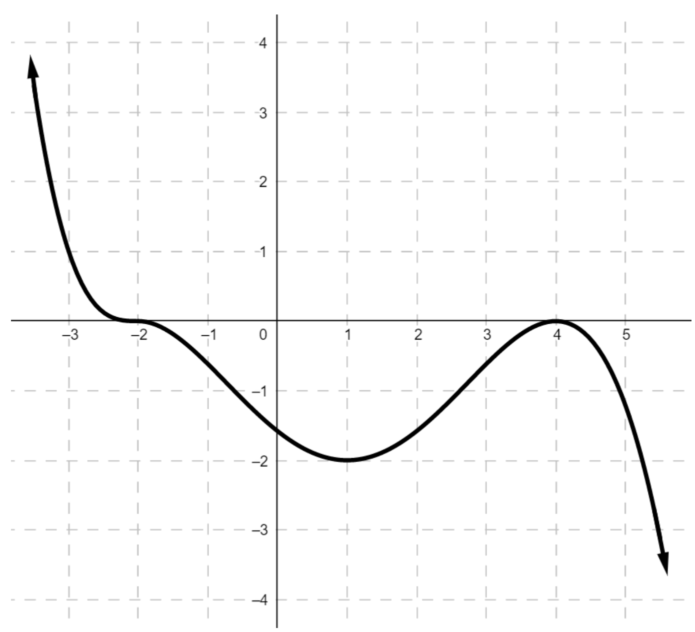
18. Given that $\int_5^1 f(x)\,dx = -2$, $\int_3^5 g(x)\,dx = 9$, and $\int_3^1 g(x)\,dx = 7$, determine the value of $\int_1^5 (4f(x) - g(x))$.

(a) -24
(b) 0
(c) 14
(d) -8
(e) 6

The graph below is the **derivative**, $f'(x)$, of a continuous function f whose domain is all real numbers. Use this graph to answer Questions 19 and 20.

19. On what interval(s) is $f(x)$ decreasing?

(a) $(-\infty, -2), (-2, 4), (4, \infty)$
(b) $(-\infty, -2)$
(c) $(-2, 4), (4, \infty)$
(d) $(-\infty, 1), (4, \infty)$
(e) $(1, 4)$



20. For what x-value(s) does f have an inflection point?

(a) $x = -2$ only
(b) $x = -2$ and $x = 4$
(c) $x = 1$ and $x = 4$
(d) $x = 1$ only
(e) $x = -2$, $x = 1$, and $x = 4$