MATH 151, FALL 2021
COMMON EXAM I - VERSION A

LAST NAME(print): ______________________________FIRST NAME(print): ______________________________

INSTRUCTOR: ______________________________

SECTION NUMBER: _______________

DIRECTIONS:

1. No calculator, cell phones, or other electronic devices may be used, and they must all be put away out of sight.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. **Be sure to *fill in your name, UIN, section number and version letter of the exam on the ScanTron form.***

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____

PART I: Multiple Choice. 3 points each

1. Find the vector $\mathbf{a}$ that has magnitude $|\mathbf{a}| = 6$ and makes an angle of $300°$ with the positive x-axis.

   (a) $3\mathbf{i} + 3\sqrt{3}\mathbf{j}$
   (b) $3\sqrt{3}\mathbf{i} + 3\mathbf{j}$
   (c) $3\mathbf{i} - 3\sqrt{3}\mathbf{j}$ ← correct
   (d) $3\sqrt{3}\mathbf{i} - 3\mathbf{j}$
   (e) $-3\sqrt{3}\mathbf{i} - 3\mathbf{j}$

2. For points $A(1,3)$, $B(-3,1)$, and $C(2,1)$, Which of the following statements is **false**?

   (a) $\overrightarrow{AB} = \langle -4, -2 \rangle$.
   (b) The magnitude of $\overrightarrow{AC}$ is $\sqrt{5}$.
   (c) The magnitude of $\overrightarrow{AB}$ is $\sqrt{20}$
   (d) $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{AC}$
   (e) $\cos\theta = \dfrac{\langle 5, 0 \rangle \cdot \langle -4, -2 \rangle}{5\sqrt{20}}$, where $\theta = \angle ABC$. ← correct

3. A force $\overrightarrow{F} = 2\mathbf{i} + 6\mathbf{j}$ moves an object from the point $P(2,2)$ to the point $Q(4,6)$. How much work is done if the force is measured in pounds and the distance is measured in feet?

   (a) 19 foot pounds
   (b) 28 foot pounds ← correct
   (c) 32 foot pounds
   (d) 45 foot pounds
   (e) 68 foot pounds

4. Which of the following vectors is parallel to the line $2x + 4y = 11$?

   (a) $\langle 4, 2 \rangle$
   (b) $\langle 2, 4 \rangle$
   (c) $\langle -4, 2 \rangle$ ← correct
   (d) $\langle -2, 4 \rangle$
   (e) $\langle 2, -4 \rangle$

5. Find a vector equation for the line which passes the point $(2, -1)$ and is perpendicular to $\langle 3, 4 \rangle$

   (a) $\mathbf{r}(t) = \langle 2 - 4t, -1 + 3t \rangle$ ← correct
   (b) $\mathbf{r}(t) = \langle 2 + 4t, -1 + 3t \rangle$
   (c) $\mathbf{r}(t) = \langle 2 + 3t, -1 + 4t \rangle$
   (d) $\mathbf{r}(t) = \langle 2 - 3t, -1 - 4t \rangle$
   (e) $\mathbf{r}(t) = \langle 1 - 4t, -2 + 3t \rangle$

6. Find the distance from the point $(1, 5)$ to the line $y = 2x + 1$.

(a) $\dfrac{2}{\sqrt{3}}$

(b) $\dfrac{6}{\sqrt{5}}$

(c) $\dfrac{9}{\sqrt{5}}$

(d) $\dfrac{2}{\sqrt{5}}$ ← correct

(e) $\dfrac{9}{\sqrt{3}}$

7. The motion of a particle is given by the vector function $\mathbf{r}(t) = \langle 2\cos t, -3\sin t \rangle$. Which of the following describes the motion of the particle as t increases?

(a) Clockwise around a circle

(b) Counterclockwise around an ellipse

(c) Counterclockwise around a circle

(d) Clockwise around an ellipse ←correct

(e) None of these

8. Find the intersection point of this pair of lines.

$$L_1(t) = \langle 1 + t,\ 2 + t \rangle \qquad\qquad L_2(s) = \langle 5 - 2s,\ 3 + s \rangle$$

(a) $(1, 2)$

(b) $(1, 5)$

(c) $(2, 3)$

(d) $(3, 4)$ ← correct

(e) $(2, 1)$

9. Simplify $\cos\left(\arcsin\left(\frac{x}{3}\right)\right)$ to an algebraic expression.

(a) $\dfrac{3}{\sqrt{9 - x^2}}$

(b) $\dfrac{3}{\sqrt{x^2 + 9}}$

(c) $\dfrac{\sqrt{x^2 + 9}}{3}$

(d) $\dfrac{3 - x}{3}$

(e) $\dfrac{\sqrt{9 - x^2}}{3}$ ← correct

10. Find the limit $\lim_{x \to -4^-} \frac{x}{x+4}$

(a) $-\frac{1}{4}$
(b) 0
(c) $\frac{1}{4}$
(d) $-\infty$
(e) ∞ ← correct

11. Evaluate $\lim_{t \to 1} \frac{1-t^2}{1-\sqrt{t}}$

(a) 1
(b) 2
(c) 3
(d) 4 ← correct
(e) Does not exist

12. Evaluate $\lim_{x \to -\infty} \frac{\sqrt{9x^2+12x-7}}{-2x+2}$

(a) $-\frac{9}{2}$
(b) $-\frac{3}{2}$
(c) 0
(d) $\frac{3}{2}$ ← correct
(e) $\frac{9}{2}$

13. Given $f(x) = \frac{1}{x}$ and $f'(x) = -\frac{1}{x^2}$, find the equation of tangent line of $f(x)$ at $x = 2$.

(a) $y - \frac{1}{2} = -\frac{1}{4}(x-2)$ ← correct
(b) $y - 2 = -\frac{1}{4}(x-2)$
(c) $y + \frac{1}{2} = -\frac{1}{4}(x-2)$
(d) $y - \frac{1}{2} = \frac{1}{4}(x-2)$
(e) $y + \frac{1}{2} = \frac{1}{4}(x-2)$

14. Find the average rate of change of $f(t) = \sqrt{2t+3}$ from $t = 1$ to $t = 3$.

(a) $3 - \sqrt{5}$

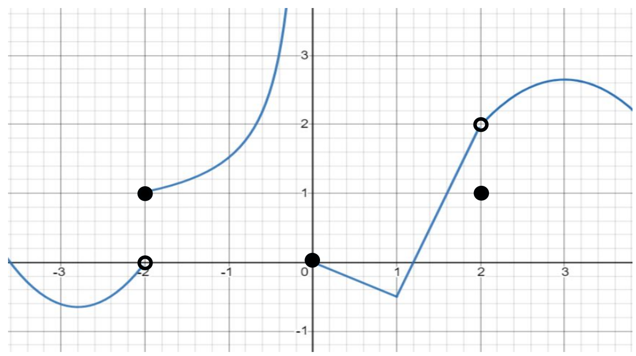
(b) $\dfrac{3+\sqrt{5}}{2}$

(c) $\dfrac{3-\sqrt{5}}{2}$ ← correct

(d) $3 + \sqrt{5}$

(e) $\dfrac{\sqrt{5}-3}{2}$

Use the graph of f to the right to answer Questions 15 and 16.



15. Which of the following statements is **false** concerning the limit of f?

(a) $\lim_{x\to -\mathbf{2}^-} f(x) = 0$

(b) $\lim_{x\to -\mathbf{2}^+} f(x) = 1$

(c) $\lim_{x\to \mathbf{0}} f(x) = 0$ ← correct

(d) $\lim_{x\to \mathbf{1}^-} f(x) = \lim_{x\to \mathbf{1}^+} f(x)$

(e) $\lim_{x\to \mathbf{2}} f(x) = 2$

16. Which of the following statements is **false** concerning the graph of f?

(a) f is continuous from the right at $x = -2$.

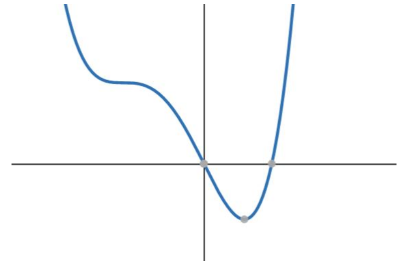
(b) f has a jump discontiuity at $x = -2$.

(c) f is continuous from the right at $x = 0$.

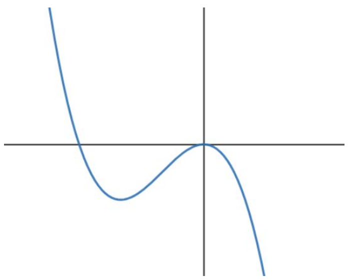
(d) f has a removable discontiuity at $x = 2$.

(e) f is continuous and differentiable at $x = 1$ ← correct

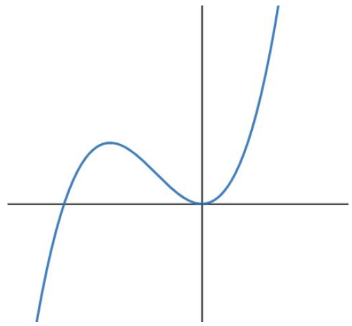
17. Consider the graph of $f(x)$ to the right.
Which of the following is the graph of the derivative, i.e., $f'(x)$?



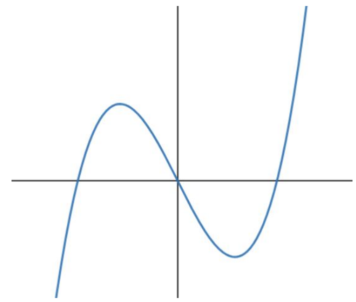
(a)



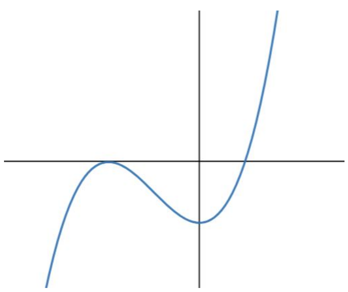
(b)



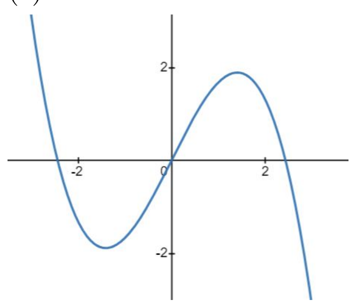
(c)



(d) $\leftarrow$ correct



(e)



18. Find the limit $\lim\limits_{x\to \mathbf{2}^-} e^{1/(x-2)}$.

(a) 0 $\leftarrow$ correct
(b) -2
(c) 2
(d) $-\infty$
(e) ∞

19. Find the horizontal and vertical asymptotes for $f(x) = \dfrac{(2-x)(3x+1)}{x^2-4}$

(a) $y = -3$, $x = -2$ $\leftarrow$ correct
(b) $y = -3$, $x = -2$, $x = 2$
(c) $y = -2$, $y = 2$, $x = -3$
(d) $y = -2$, $x = -3$
(e) $y = 3$, $x = -2$

20. Which of the following intervals contains a root to the equation $x^3 + 2x^2 - 42 = 0$?

(a) $(-2, 0)$
(b) $(-1, 0)$
(c) $(0, 1)$
(d) $(1, 2)$
(e) $(2, 3)$ $\leftarrow$ correct

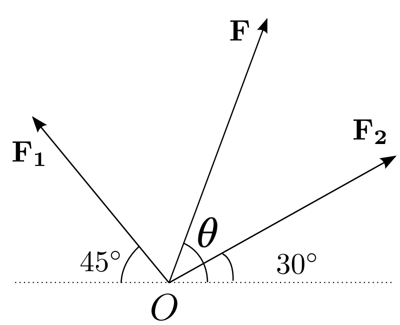
PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

21. (10 points) Two forces act on an object as in the diagram below. $\mathbf{F_1}$ has a magnitude of 20 pounds and $\mathbf{F_2}$ has a magnitudes of 36 lbs.

(a) Find the vectors $\mathbf{F_1}$, $\mathbf{F_2}$, and the resultant force $\mathbf{F}$. Your answers do not need to be simplified, but all trigonometric expressions which can be evaluated must be.

$$\mathbf{F_1} = |\mathbf{F_1}|\langle \cos 135°, \sin 135° \rangle = 20\langle -\cos 45°, \sin 45° \rangle$$
$$= 20\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle = \langle -10\sqrt{2}, 10\sqrt{2} \rangle$$



$$\mathbf{F_2} = |\mathbf{F_2}|\langle \cos 30°, \sin 30° \rangle = 36\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle = \langle 18\sqrt{3}, 18 \rangle$$

$$\mathbf{F} = \mathbf{F_1} + \mathbf{F_2} = \langle -10\sqrt{2}, 10\sqrt{2} \rangle + \langle 18\sqrt{3}, 18 \rangle$$
$$= \langle -10\sqrt{2} + 18\sqrt{3}, 10\sqrt{2} + 18 \rangle$$

(b) Find the resultant angle θ as shown in the diagram. Leave your answer in terms of an inverse trigonometric expression.

Since $\tan\theta = \dfrac{10\sqrt{2} + 18}{-10\sqrt{2} + 18\sqrt{3}}$ we have

$$\theta = \tan^{-1}\left(\frac{10\sqrt{2} + 18}{-10\sqrt{2} + 18\sqrt{3}}\right).$$

22. (15 points) Evaluate these limits. Do not use the L'Hopital method.

(a) $\lim\limits_{x \to -4} \dfrac{x^2+2x-8}{x^2+x-12}$

$$\lim_{x \to -4} \frac{x^2+2x-8}{x^2+x-12} = \lim_{x \to -4} \frac{(x-2)(x+4)}{(x-3)(x+4)} = \lim_{x \to -4} \frac{x-2}{x-3} = \frac{-6}{-7} = \frac{6}{7}$$

(b) $\lim\limits_{x \to 3^-} \dfrac{x^2-9}{|x-3|}$

Since $x < 3$, $|x-3| = -(x-3)$. Thus we have

$$\lim_{x \to 3^-} \frac{x^2-9}{|x-3|} = \lim_{x \to 3^-} \frac{x^2-9}{-(x-3)} = \lim_{x \to 3^-} \frac{(x-3)(x+3)}{-(x-3)} = \lim_{x \to 3^-} -(3+x) = -6.$$

(c) $\lim\limits_{x \to 2} \dfrac{\sqrt{7x+2}-4}{x-2}$

$$\lim_{x \to 2} \frac{\sqrt{7x+2}-4}{x-2} = \lim_{x \to 2} \frac{(\sqrt{7x+2}-4)(\sqrt{7x+2}+4)}{(x-2)(\sqrt{7x+2}+4)} = \lim_{x \to 2} \frac{(7x+2)-16}{(x-2)(\sqrt{7x+2}+4)}$$

$$= \lim_{x \to 2} \frac{7x-14}{(x-2)(\sqrt{7x+2}+4)} = \lim_{x \to 2} \frac{7(x-2)}{(x-2)(\sqrt{7x+2}+4)}$$

$$= \lim_{x \to 2} \frac{7}{(\sqrt{7x+2}+4)} = \frac{7}{(\sqrt{7 \cdot 2+2}+4)} = \frac{7}{4+4} = \frac{7}{8}.$$

23. (7 points) Let A and B be constants. Consider the function

$$g(x) = \begin{cases} 4x + A, & \text{if } x < 2 \\ B & \text{if } x = 2 \\ x^2 - Ax + 1, & \text{if } x > 2 \end{cases}$$

(a) Determine the value of A for which $\lim_{x\to 2} g(x)$ exists.

If $\lim_{x\to 2} g(x)$ exists, two one sided limits $\lim_{x\to 2^-} g(x)$ and $\lim_{x\to 2^+} g(x)$ must agree. We have

$$\lim_{x\to 2^-} 4x + A = \lim_{x\to 2^+} x^2 - Ax + 1.$$

This implies $8 + A = 4 - 2A + 1$ or $A = -1$.

(b) Determine the value of B for which $g(x)$ is continuous everywhere.

In order for $g(x)$ to be continuous, $\lim_{x\to 2} g(x) = g(2)$. With $A = -1$ from (a) above, we have

$$\lim_{x\to 2^-} 4x + A = \lim_{x\to 2^-} 4x - 1 = 7 = B$$

24. (8 points) Use the definition of the derivative to find $f'(x)$ for $f(x) = \frac{2}{x-3}$. No points will be given for any shortcut formulas used.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{2}{x+h-3} - \frac{2}{x-3}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2(x-3) - 2(x+h-3)}{(x+h-3)(x-3)}}{h} = \lim_{h\to 0} \frac{\frac{-2h}{(x+h-3)(x-3)}}{h}$$

$$= \lim_{h\to 0} \frac{-2h}{h(x+h-3)(x-3)} = \lim_{h\to 0} \frac{-2}{(x+h-3)(x-3)} = \frac{-2}{(x-3)^2}$$