

MATH 151, SPRING 2022
COMMON EXAM II - VERSION **A**

LAST NAME(print): _____ FIRST NAME(print): _____

INSTRUCTOR: _____

SECTION NUMBER: _____

DIRECTIONS:

1. No calculator, cell phones, or other electronic devices may be used, and they must all be put away out of sight.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. **Be sure to fill in your name, UIN, section number and version letter of the exam on the ScanTron form.**

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: _____

PART I: Multiple Choice. 3 points each

1. Find the equation of the tangent line to the graph of $f(x) = \frac{2x}{3x+1}$ at $x = 1$.

- (a) $y - \frac{1}{2} = -\frac{1}{8}(x - 1)$
- (b) $y - \frac{1}{2} = \frac{7}{8}(x - 1)$
- (c) $y - \frac{1}{2} = -\frac{2}{3}(x - 1)$
- (d) $y - \frac{1}{2} = \frac{1}{8}(x - 1)$ ← correct
- (e) $y - \frac{1}{2} = -\frac{7}{8}(x - 1)$

2. Find the equation of the tangent line to the graph of $x = 3t^2 - 2t + 1$, $y = t^2 + 1$ at the point where $t = 1$.

- (a) $y = \frac{1}{2}x + 1$ ← correct
- (b) $y = \frac{1}{2}x - 1$
- (c) $y = 2x - 2$
- (d) $y = x$
- (e) None of these

3. Find the slope of the tangent line to the curve $x^2 - y^2 = 16$ at the point $(-5, 3)$.

- (a) $-\frac{5}{3}$ ← correct
- (b) $\frac{5}{3}$
- (c) $-\frac{5}{4}$
- (d) $\frac{4}{5}$
- (e) $\frac{3}{4}$

4. Determine where the function f below is NOT differentiable.

$$f(x) = \begin{cases} \sin x & x < 0 \\ -x^3 + x & 0 \leq x < 1 \\ x^2 - 1 & 1 \leq x \end{cases}$$

- (a) $x = 0$ only
- (b) $x = 1$ only ← correct
- (c) $x = 0$ and $x = 1$
- (d) f is differentiable everywhere
- (e) f is not differentiable anywhere

Suppose f and g are differentiable functions which satisfy the following condition. Use the following table for Problems 5 and 6.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	-2	1	1	1
1	-1	3	3	0

table updated

5. Let $u(x) = \frac{f(x)}{g(x)}$. Find $u'(1)$.

- (a) -1
- (b) 0
- (c) 1 ← correct
- (d) 2
- (e) 3

6. Let $v(x) = g(f(x))$. Find $v'(1)$. function updated

- (a) -2
- (b) -1
- (c) 0
- (d) 3 ← correct
- (e) 5

7. Find the derivative of the function $f(x) = \arcsin(e^{5x})$.

(a) $\frac{5e^{5x}}{1 + e^{10x}}$

(b) $\frac{1}{\sqrt{1 - e^{10x}}}$

(c) $-\frac{5e^{5x}}{\sqrt{1 - e^{10x}}}$

(d) $-\frac{5e^{5x}}{1 + e^{10x}}$

(e) $\frac{5e^{5x}}{\sqrt{1 - e^{10x}}}$ ← correct

8. Find all point(s) on the curve parametrized by $x = t^2 - 2t - 3$, $y = t^3 - 3t^2$ where the tangent line is horizontal.

(a) $(-4, -2)$

(b) $(0, 0)$ and $(-3, 0)$

(c) $(-3, -4)$ and $(-3, 0)$ ←correct

(d) $(0, 0)$ and $(0, -4)$

(e) None of these

9. Find all point(s) on the curve parametrized by $x = t^2 - 2t - 3$, $y = t^3 - 3t^2$ where the tangent line is vertical.

(a) $(-4, -2)$ ←correct

(b) $(0, 0)$ and $(-3, 0)$

(c) $(-3, -4)$ and $(-3, 0)$

(d) $(0, 0)$ and $(0, -4)$

(e) None of these

10. Find $f'(0)$ for $f(x) = 2^{(3^x)}$.

- (a) $\ln 2 \cdot \ln 3$
- (b) $2 \ln 2 \cdot \ln 3$ ← correct
- (c) $3 \ln 2 \cdot \ln 3$
- (d) $8 \ln 2 \cdot \ln 3$
- (e) $24 \ln 2 \cdot \ln 3$

11. Use the linearization of $f(x) = \sqrt{x}$ at $x = 9$ to estimate the value of $\sqrt{9.1}$.

- (a) $\frac{179}{60}$
- (b) $\frac{19}{6}$
- (c) $\frac{17}{6}$
- (d) $\frac{31}{30}$
- (e) $\frac{181}{60}$ ← correct

12. The length of a rectangle is increasing at a rate of 5 cm/sec and its width is decreasing at a rate of 4 cm/sec. When the length is 20 cm and the width is 10 cm, what is the rate of change of the area of the rectangle?

- (a) $30 \text{ cm}^2/\text{sec}$
- (b) $-30 \text{ cm}^2/\text{sec}$ ← correct
- (c) $60 \text{ cm}^2/\text{sec}$
- (d) $-60 \text{ cm}^2/\text{sec}$
- (e) $130 \text{ cm}^2/\text{sec}$

13. Find the 2021st derivative of $f(x) = 2 \cos(2x)$

- (a) $-2^{2021} \sin(2x)$
- (b) $2^{2022} \sin(2x)$
- (c) $2^{2022} \cos(2x)$
- (d) $-2^{2021} \cos(2x)$
- (e) $-2^{2022} \sin(2x)$ ← correct

14. The position of a particle is given by the vector function $\mathbf{r}(t) = \langle te^t, t^3 \rangle$. Find the acceleration vector of the particle at time $t = 1$.

- (a) $\langle e, 6 \rangle$
- (b) $\langle 2e, 6 \rangle$
- (c) $\langle 2e, 3 \rangle$
- (d) $\langle e, 3 \rangle$
- (e) $\langle 3e, 6 \rangle$ ← correct

15. The radius of a circle was measured to be 5 ft with a maximum possible error of 0.2 ft. Use differentials to estimate the maximum possible error in the calculated area of the circle.

- (a) 10π
- (b) 2π ← correct
- (c) 0.9π
- (d) 0.09π
- (e) $\frac{2\pi}{25}$

16. A ball is tossed in the air, and the height of the ball at time t seconds is given by $h(t) = 25 + 10t - t^2$, where $h(t)$ is measured in feet from the ground. Find the maximum height H of the ball and the time T when it hits the maximum height.

- (a) $H = 25, T = 5$
- (b) $H = 50, T = 5$ ← correct
- (c) $H = 50, T = 10$
- (d) $H = 25, T = 10$
- (e) $H = 30, T = 5$

17. A particle moves according to the equation of motion $s(t) = t^2 - 2t + 3$ where $s(t)$ is measured in meter and t is measured in seconds. Find the total distance traveled in the first 3 seconds.

- (a) 2 m
- (b) 3 m
- (c) 4 m
- (d) 5 m ← correct
- (e) 6 m

18. A bacteria culture starts with 2 million bacteria and the population triples every 30 minutes. Find the number of bacteria after 90 minutes.

- (a) 16 million
- (b) 18 million
- (c) 30 million
- (d) 54 million ← correct
- (e) 72 million

19. Find $h''(1)$ if $h(x) = e^{-x^2}$

(a) $\frac{4}{e}$

(b) $\frac{2}{e}$ ← correct

(c) $-\frac{2}{e}$

(d) $\frac{1}{e}$

(e) $-\frac{4}{e}$

20. Find $f'(x)$ for $f(x) = \ln\left(\frac{\sqrt{x^6+1}}{\sec^{10} x}\right)$ [HINT: Use properties of logarithms.]

(a) $\frac{1}{12x^5} - \frac{10}{\sec x \tan x}$

(b) $\frac{1}{12x^5} - \frac{1}{10 \sec x \tan x}$

(c) $\frac{3x^5}{x^6+1} - \frac{10 \tan x}{\sec x}$

(d) $\frac{3x^5}{x^6+1} - 10 \tan x$ ← correct

(e) $\frac{3x^5}{x^6+1} - \tan x$

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

21. (12 points) Find $\frac{dy}{dx}$ for the following functions. **Do not simplify after taking the derivative.**

(a) $y = \sec^5(4x^3 + \pi^2)$

Sol. Apply the Power Rule and Chain Rule to have

$$\begin{aligned}\frac{dy}{dx} &= 5 \sec^4(4x^3 + \pi^2) \left(\sec(4x^3 + \pi^2) \right)' \\ &= 5 \sec^4(4x^3 + \pi^2) \sec(4x^3 + \pi^2) \tan(4x^3 + \pi^2) (4x^3 + \pi^2)' \\ &= 5 \sec^4(4x^3 + \pi^2) \sec(4x^3 + \pi^2) \tan(4x^3 + \pi^2) (12x^2)\end{aligned}$$

(b) $y = \frac{2^x \tan x}{x^3}$

Sol. Apply the Quotient Rule and Chain Rule to have

$$\begin{aligned}\frac{dy}{dx} &= \frac{(2^x \tan x)'(x^3) - 2^x \tan x (x^3)'}{(x^3)^2} \\ &= \frac{(2^x \ln 2 \tan x + 2^x \sec^2 x)(x^3) - 2^x \tan x (3x^2)}{x^6}\end{aligned}$$

Alternatively, one can apply Logarithmic Differentiation.

$$\begin{aligned}\ln y &= \ln \left(\frac{2^x \tan x}{x^3} \right) = x \ln 2 + \ln \tan x - 3 \ln x \\ \Rightarrow \frac{y'}{y} &= \ln 2 + \frac{\sec^2 x}{\tan x} - \frac{3}{x} \\ \Rightarrow y' &= \left(\ln 2 + \frac{\sec^2 x}{\tan x} - \frac{3}{x} \right) \frac{2^x \tan x}{x^3}\end{aligned}$$

22. (6 points) Find $\frac{dy}{dx}$.

$$y = e^{5xy}$$

Sol. Apply Implicit Differentiation to have

$$\begin{aligned}y' &= e^{5xy} (5xy)' = e^{5xy} (5y + 5xy') = e^{5xy} 5y + e^{5xy} 5xy' \\ \Rightarrow y' - e^{5xy} 5xy' &= e^{5xy} 5y \\ \Rightarrow (1 - e^{5xy} 5x)y' &= e^{5xy} 5y\end{aligned}$$

So we have

$$\frac{dy}{dx} = \frac{5e^{5xy}y}{1 - 5e^{5xy}x}$$

23. (10 points) Find $\frac{dy}{dx}$. **Your answer must be a function in x only.**

$$y = (\cos(4x))^{\sqrt{x}}$$

Sol. Take \ln of $y = (\cos 4x)^{\sqrt{x}}$ to have

$$\ln y = \ln (\cos 4x)^{\sqrt{x}} = \sqrt{x} \cdot \ln(\cos 4x)$$

Apply Implicit Differentiation to have

$$\begin{aligned}\frac{y'}{y} &= (\sqrt{x})' \cdot \ln(\cos 4x) + \sqrt{x} \cdot (\ln(\cos 4x))' \\ &= \left(\frac{1}{2}x^{-1/2}\right) \ln(\cos 4x) + \sqrt{x} \cdot \frac{1}{\cos 4x} (-\sin 4x)4\end{aligned}$$

Thus we have

$$y' = \left(\frac{1}{2\sqrt{x}} \ln(\cos 4x) + \sqrt{x} \cdot \frac{-4 \sin 4x}{\cos 4x}\right) (\cos 4x)^{\sqrt{x}}$$

Acceptible answers include

$$y = \left(\frac{1}{2\sqrt{x}} \ln(\cos 4x) - 4\sqrt{x} \tan 4x\right) (\cos 4x)^{\sqrt{x}}$$

24. (12 points) Water is pumped into an inverted conical tank at a constant rate of $3 \text{ m}^3/\text{min}$. The tank has a height of 10 m, and the diameter across the top is 10 m. How fast is the water level rising when the water is 2 m deep? (The volume of a cone is $V = \frac{1}{3}\pi r^2 h$).

Sol. From the condition of height $h = h(t)$ and the radius $r = r(t)$, we have

$$h : 2r = 10 : 10 \quad \Rightarrow \quad r(t) = \frac{h(t)}{2}$$

The volume can be expressed as

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$$

The rate of change in volume $V = V(h)$ is

$$\frac{dV}{dt} = \frac{\pi}{12} \left(h^3(t)\right)' = \frac{\pi}{12} 3h^2 h'(t) = \frac{\pi h^2}{4} \cdot \frac{dh}{dt}$$

The condition implies $h = 2$, $\frac{dV}{dt} = 3$. Thus we have

$$3 = \frac{\pi 2^2}{4} \cdot \frac{dh}{dt} \quad \Rightarrow \quad \frac{dh}{dt} = \frac{3}{\pi} \text{ (m/min)}.$$

DO NOT WRITE IN THIS TABLE.

Question	Points Awarded	Points
1-20		60
21		12
22		6
23		10
24		12
		100