MATH 151, FALL 2021
COMMON EXAM II - VERSION B

LAST NAME(print): ______________________ FIRST NAME(print): ______________________

INSTRUCTOR: ______________________

SECTION NUMBER: ____________

DIRECTIONS:

1. No calculator, cell phones, or other electronic devices may be used, and they must all be put away out of sight.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. **Be sure to *fill in your name, UIN, section number and version letter of the exam on the ScanTron form.***

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________________

PART I: Multiple Choice. 3 points each

Suppose f and g are differentiable functions which satisfy the following condition. Use the following table for Questions 1 and 2.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	-1	3	5	0
1	1	2	-1	2

1. Find $H'(1)$ for $H(x) = \dfrac{f(g(x))}{x^2}$.

   (a) 0
   (b) 2
   (c) 4
   (d) 6
   (e) 8

2. Find $h'(1)$ for $h(x) = x^2 f(g(x))$.

   (a) 1
   (b) 4
   (c) 6
   (d) 8
   (e) 12

3. Which of the statements is true about $f(x)$?

$$f(x) = \begin{cases} e^x + 4, & x < 0 \\ x^3 + x + 5, & 0 \le x \le 2 \\ 10x - 5, & 2 < x \end{cases}$$

   (a) f is continuous but not differentiable at both $x = 0$ and $x = 2$.
   (b) f is continuous but not differentiable at $x = 0$; f is differentiable at $x = 2$.
   (c) f is differentiable at both $x = 0$ and $x = 2$.
   (d) f is differentiable at $x = 0$; f is continuous but not differentiable at $x = 2$.
   (e) f is not continuous at $x = 0$ or $x = 2$.

4. Find $f''(x)$ for $f(x) = e^{1/x}$.

(a) $\left(\frac{2}{x^3} - \frac{1}{x^2}\right)e^{1/x}$

(b) $\left(\frac{2}{x^3} - \frac{1}{x^3}\right)e^{1/x}$

(c) $\left(\frac{1}{x^2}\right)e^{1/x}$

(d) $\left(-\frac{2}{x^5}\right)e^{1/x}$

(e) $\left(\frac{2}{x^3} + \frac{1}{x^4}\right)e^{1/x}$

5. Find the derivative of the function $f(x) = \arcsin\left(3^{6x}\right)$

(a) $\dfrac{6 \cdot 3^{6x}}{\sqrt{1 - 3^{12x}}}$

(b) $-\dfrac{6 \cdot 3^{6x}}{\sqrt{1 - 3^{12x}}}$

(c) $\dfrac{6 \ln 3 \cdot 3^{6x}}{1 + 3^{12x}}$

(d) $-\dfrac{6 \ln 3 \cdot 3^{6x}}{\sqrt{1 - 3^{12x}}}$

(e) $\dfrac{6 \ln 3 \cdot 3^{6x}}{\sqrt{1 - 3^{12x}}}$

6. Find the 2021^{th} derivative of $f(x) = 2xe^{-x}$.

(a) $e^{-x}(-x + 2021)$

(b) $e^{-x}(x - 2021)$

(c) $2e^{-x}(x - 2021)$

(d) $2e^{-x}(-x + 2021)$

(e) $2^{2021}e^{-x}(-x + 2021)$

7. Find all the value(s) of x on the interval $[0, 2\pi]$ for which the tangent line to the graph of $f(x) = \sin^2 x - \cos x$ is horizontal.

   (a) $0, \pi, 2\pi$

   (b) $0, \pi, 2\pi, \frac{\pi}{6}, \frac{5\pi}{6}$

   (c) $0, \pi, 2\pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

   (d) $0, \pi, 2\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

   (e) $0, \pi, 2\pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

8. A ball is thrown vertically upward with a velocity of 40 feet per second and the height, s, of the ball at time t seconds is given by $s(t) = 40t - 8t^2$. What is the velocity of the ball when it is 48 feet above the ground on its way up?

   (a) 12 ft/sec

   (b) 24 ft/sec

   (c) 32 ft/sec

   (d) 54 ft/sec

   (e) 8 ft/sec

9. Let $g(x) = f(x^2 + 1)$. Given the table of values below for f and f', find the equation of the tangent line to $g(x)$ at $x = 1$.

x	$f(x)$	$f'(x)$
1	3	4
2	5	6
3	1	2

   (a) $y - 5 = -2(x - 1)$

   (b) $y - 3 = 4(x - 1)$

   (c) $y - 3 = 6(x - 1)$

   (d) $y - 5 = 6(x - 1)$

   (e) $y - 5 = 12(x - 1)$

10. Find the slope of the tangent line to the curve $y^3 - xy = 2x + 4$ at the point $(1, 2)$.

(a) $\frac{1}{3}$

(b) 8

(c) $\frac{4}{11}$

(d) $\frac{22}{11}$

(e) $\frac{5}{3}$

11. Find $f'(x)$ for $f(x) = \ln\left(\frac{\sqrt{x^4 - 3}}{\sec^2 x}\right)$

(a) $\frac{1}{8x^3} - \frac{2}{\sec x \tan x}$

(b) $\frac{1}{8x^3} - \frac{1}{2\sec x \tan x}$

(c) $\frac{2x^3}{x^4 - 3} - 2\tan x$

(d) $\frac{2x^3}{x^4 - 3} - \frac{2\tan x}{\sec x}$

(e) $\frac{2x^3}{x^4 - 3} - \tan x$

12. Find the derivative of $f(x) = \log_{10}(x^5 + 5^x)$.

(a) $\frac{5x^4 + 5^x}{(x^5 + 5^x)\ln 10}$

(b) $\frac{5x^4}{(x^5 + 5^x)\ln 10}$

(c) $\frac{5x^4 + 5^x \ln 5}{x^5 + 5^x}$

(d) $\frac{5x^4 + 5^x \ln 5}{(x^5 + 5^x)\ln 10}$

(e) $\frac{5x^4 + 5^x}{x^5 + 5^x}$

13. Which of the following vectors is tangent to the curve $\mathbf{r}(t) = \left\langle \sqrt{t^2+1}, t \right\rangle$ when $t = \sqrt{3}$?

(a) $\left\langle \frac{\sqrt{3}}{4}, 1 \right\rangle$

(b) $\left\langle \frac{\sqrt{3}}{2}, 1 \right\rangle$

(c) $\left\langle \frac{1}{2}, 1 \right\rangle$

(d) $\left\langle \frac{1}{\sqrt{5}}, 1 \right\rangle$

(e) $\left\langle \frac{2}{\sqrt{5}}, 1 \right\rangle$

14. Let $\mathbf{r}(t) = \langle \sin(2t), \cos t \rangle$. For which of the following points does the curve have a vertical tangent line?

(a) $P = \left(1, \frac{\sqrt{2}}{2}\right)$

(b) $P = \left(1, \frac{\sqrt{3}}{2}\right)$

(c) $P = \left(\frac{\sqrt{3}}{2}, 1\right)$

(d) $P = \left(\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$

(e) $P = (0, 1)$

15. Find the slope of the tangent line at the point $(2, 0)$ for the following parametric curve

$$x(t) = t^4 + 1, \quad y(t) = \cos\left(\frac{\pi t}{2}\right)$$

(a) $-\frac{8}{\pi}$

(b) $-\frac{1}{4}$

(c) $-\frac{\pi}{8}$

(d) 0

(e) $-\frac{\pi}{4}$

16. The position function of an object moving along a straight line is given by $s(t) = t^4 - 2t^2 + 1$, where position is measured in feet and time in seconds. Find the total distance traveled by the object during the first two seconds.

(a) 8

(b) 9

(c) 10

(d) 11

(e) 12

17. A sample of a radioactive substance decayed to $\dfrac{1}{5}$ of its original amount after 9 years. What is the half-life of the substance? The half-life is the amount of time needed to decay to half of its original amount.

(a) $\dfrac{\ln\frac{1}{5}}{9\ln\frac{1}{2}}$ years

(b) $-\dfrac{9\ln\frac{1}{5}}{\ln\frac{1}{2}}$ years

(c) $\dfrac{9\ln\frac{1}{5}}{\ln\frac{1}{2}}$

(d) $\dfrac{9\ln\frac{1}{2}}{\ln\frac{1}{5}}$ years

(e) $-\dfrac{9\ln\frac{1}{2}}{\ln\frac{1}{5}}$ years

18. A cubic block of ice (which remains in the shape of a cube) is melting so that its volume is decreasing at a rate of 4 cm^3/min. How fast is the length of a side changing (in cm/min) when the sides are 10 cm?

(a) $\dfrac{4}{300}$

(b) $\dfrac{1}{1200}$

(c) 0

(d) $-\dfrac{1}{1200}$

(e) $-\dfrac{4}{300}$

19. Use a linear approximation (or differentials) at $x = 8$ to approximate the value of $\sqrt[3]{8.1}$.

(a) $\dfrac{241}{120}$

(b) $\dfrac{121}{60}$

(c) $\dfrac{481}{240}$

(d) $\dfrac{61}{30}$

(e) $\dfrac{961}{480}$

20. The radius of a circle is measured to be 1 meter with a possible error of 0.04m. Use differentials or linear approximation to estimate the maximum possible error in the area of the circle.

(a) 0.08π

(b) 0.06π

(c) 0.04π

(d) 0.10π

(e) 0.05π

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

21. (12 points) Find $\dfrac{dy}{dx}$ for the following functions. **Do not simplify after taking the derivative**.

    (a) $y = \tan^4\big(\cos(3x)\big)$

    (b) $y = \dfrac{e^{2x}\tan x}{x^3 + \pi^3}$

22. (7 points) Find $\dfrac{dy}{dx}$. Your answer must be a function in x only.

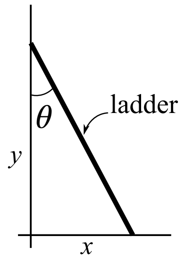
$y = \big(\sin(3x)\big)^{2\sqrt{x}}$

23. (8 points) There are two lines tangent to the parabola $y = 2x^2$ that pass through the point $(2, -10)$.

(a) Find the x-coordinates where these tangent lines touch the parabola.

(b) Find equations of the two tangent lines.

24. (13 points) A ladder 10 feet long is leaning against a vertical wall. The base of the ladder is pulled away from the wall at a rate of 3 ft/s.



(a) How fast is the top of the ladder sliding down the wall when the base is 8 feet from the wall?

(b) Find the rate at which the angle θ betweeen the ladder and the wall is changing when the base of the ladder is 8 feet from the wall.

DO NOT WRITE IN THIS TABLE.

Question	Points Awarded	Points
1-20		60
21		10
22		15
23		7
24		8
		100