

MATH 151, SPRING 2022
COMMON EXAM II - VERSION **B**

LAST NAME(print): _____ FIRST NAME(print): _____

INSTRUCTOR: _____

SECTION NUMBER: _____

DIRECTIONS:

1. No calculator, cell phones, or other electronic devices may be used, and they must all be put away out of sight.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. **Be sure to fill in your name, UIN, section number and version letter of the exam on the ScanTron form.**

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: _____

PART I: Multiple Choice. 3 points each

1. Determine where the function f below is NOT differentiable.

$$f(x) = \begin{cases} \sin x & x < 0 \\ -x^3 + x & 0 \leq x < 1 \\ x^2 - 1 & 1 \leq x \end{cases}$$

- (a) f is differentiable everywhere
(b) $x = 0$ only
(c) $x = 1$ only
(d) $x = 0$ and $x = 1$
(e) f is not differentiable anywhere
2. Find the equation of the tangent line to the graph of $f(x) = \frac{2x}{3x+1}$ at $x = 1$.

- (a) $y - \frac{1}{2} = -\frac{1}{8}(x - 1)$
(b) $y - \frac{1}{2} = \frac{1}{8}(x - 1)$
(c) $y - \frac{1}{2} = \frac{7}{8}(x - 1)$
(d) $y - \frac{1}{2} = -\frac{2}{3}(x - 1)$
(e) $y - \frac{1}{2} = -\frac{7}{8}(x - 1)$

3. Find the equation of the tangent line to the graph of $x = 3t^2 - 2t + 1$, $y = t^2 + 1$ at the point where $t = 1$.

- (a) $y = \frac{1}{2}x - 1$
(b) $y = 2x - 2$
(c) $y = \frac{1}{2}x + 1$
(d) $y = x$
(e) None of these

Suppose f and g are differentiable functions which satisfy the following condition. Use the following table for Problems 4 and 5.

| x | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|-----|--------|---------|--------|---------|
| -1 | -2 | 1 | 1 | 1 |
| 1 | -1 | 3 | 3 | 0 |

4. Let $u(x) = \frac{f(x)}{g(x)}$. Find $u'(1)$.

- (a) -1
- (b) 3
- (c) 0
- (d) 1
- (e) 2

5. Let $v(x) = g(f(x))$. Find $v'(1)$.

- (a) 0
- (b) -2
- (c) -1
- (d) 3
- (e) 5

6. Find the slope of the tangent line to the curve $x^2 - y^2 = 16$ at the point $(-5, 3)$.

- (a) $\frac{5}{3}$
- (b) $-\frac{5}{4}$
- (c) $\frac{4}{5}$
- (d) $\frac{3}{4}$
- (e) $-\frac{5}{3}$

7. The length of a rectangle is increasing at a rate of 5 cm/sec and its width is decreasing at a rate of 4 cm/sec. When the length is 20 cm and the width is 10 cm, what is the rate of change of the area of the rectangle?

- (a) $-30 \text{ cm}^2/\text{sec}$
- (b) $30 \text{ cm}^2/\text{sec}$
- (c) $60 \text{ cm}^2/\text{sec}$
- (d) $-60 \text{ cm}^2/\text{sec}$
- (e) $130 \text{ cm}^2/\text{sec}$

8. Find all point(s) on the curve parametrized by $x = t^2 - 2t - 3$, $y = t^3 - 3t^2$ where the tangent line is horizontal.

- (a) $(-4, -2)$
- (b) $(-3, -4)$ and $(-3, 0)$
- (c) $(0, 0)$ and $(-3, 0)$
- (d) $(0, 0)$ and $(0, -4)$
- (e) None of these

9. Find all point(s) on the curve parametrized by $x = t^2 - 2t - 3$, $y = t^3 - 3t^2$ where the tangent line is vertical.

- (a) $(0, 0)$ and $(-3, 0)$
- (b) $(-3, -4)$ and $(-3, 0)$
- (c) $(-4, -2)$
- (d) $(0, 0)$ and $(0, -4)$
- (e) None of these

10. Find the derivative of the function $f(x) = \arcsin(e^{5x})$.

(a) $\frac{5e^{5x}}{\sqrt{1 - e^{10x}}}$

(b) $\frac{5e^{5x}}{1 + e^{10x}}$

(c) $\frac{1}{\sqrt{1 - e^{10x}}}$

(d) $-\frac{5e^{5x}}{\sqrt{1 - e^{10x}}}$

(e) $-\frac{5e^{5x}}{1 + e^{10x}}$

11. Find $f'(0)$ for $f(x) = 2^{(3^x)}$.

(a) $\ln 2 \cdot \ln 3$

(b) $24 \ln 2 \cdot \ln 3$

(c) $3 \ln 2 \cdot \ln 3$

(d) $8 \ln 2 \cdot \ln 3$

(e) $2 \ln 2 \cdot \ln 3$

12. Use the linearization of $f(x) = \sqrt{x}$ at $x = 9$ to estimate the value of $\sqrt{9.1}$.

(a) $\frac{179}{60}$

(b) $\frac{19}{6}$

(c) $\frac{17}{6}$

(d) $\frac{181}{60}$

(e) $\frac{31}{30}$

13. The radius of a circle was measured to be 5 ft with a maximum possible error of 0.2 ft. Use differentials to estimate the maximum possible error in the calculated area of the circle.

- (a) $\frac{2\pi}{25}$
- (b) 10π
- (c) 0.9π
- (d) 0.09π
- (e) 2π

14. Find the 2021st derivative of $f(x) = 2 \cos(2x)$

- (a) $-2^{2021} \sin(2x)$
- (b) $-2^{2022} \sin(2x)$
- (c) $2^{2022} \sin(2x)$
- (d) $2^{2022} \cos(2x)$
- (e) $-2^{2021} \cos(2x)$

15. The position of a particle is given by the vector function $\mathbf{r}(t) = \langle te^t, t^3 \rangle$. Find the acceleration vector of the particle at time $t = 1$.

- (a) $\langle e, 6 \rangle$
- (b) $\langle 2e, 6 \rangle$
- (c) $\langle 2e, 3 \rangle$
- (d) $\langle 3e, 6 \rangle$
- (e) $\langle e, 3 \rangle$

16. Find $f'(x)$ for $f(x) = \ln\left(\frac{\sqrt{x^6+1}}{\sec^{10}x}\right)$ [HINT: Use properties of logarithms.]

- (a) $\frac{1}{12x^5} - \frac{10}{\sec x \tan x}$
- (b) $\frac{1}{12x^5} - \frac{1}{10 \sec x \tan x}$
- (c) $\frac{3x^5}{x^6+1} - \frac{10 \tan}{\sec x}$
- (d) $\frac{3x^5}{x^6+1} - \tan x$
- (e) $\frac{3x^5}{x^6+1} - 10 \tan x$

17. A ball is tossed in the air, and the height of the ball at time t seconds is given by $h(t) = 25 + 10t - t^2$, where $h(t)$ is measured in feet from the ground. Find the maximum height H of the ball and the time T when it hits the maximum height.

- (a) $H = 25, T = 5$
- (b) $H = 25, T = 10$
- (c) $H = 50, T = 10$
- (d) $H = 50, T = 5$
- (e) $H = 30, T = 5$

18. A particle moves according to the equation of motion $s(t) = t^2 - 2t + 3$ where $s(t)$ is measured in meter and t is measured in seconds. Find the total distance traveled in the first 3 seconds.

- (a) 6 m
- (b) 5 m
- (c) 4 m
- (d) 3 m
- (e) 2 m

19. A bacteria culture starts with 2 million bacteria and the population triples every 30 minutes. Find the number of bacteria after 90 minutes.

- (a) 16 million
- (b) 18 million
- (c) 30 million
- (d) 54 million
- (e) 72 million

20. Find $h''(1)$ if $h(x) = e^{-x^2}$

- (a) $\frac{4}{e}$
- (b) $-\frac{2}{e}$
- (c) $\frac{2}{e}$
- (d) $\frac{1}{e}$
- (e) $-\frac{4}{e}$

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

21. (12 points) Find $\frac{dy}{dx}$ for the following functions. **Do not simplify after taking the derivative.**

(a) $y = \sec^3(4x^5 + \pi^2)$

(b) $y = \frac{3^x \tan x}{x^4}$

22. (6 points) Find $\frac{dy}{dx}$ for the equation $y = e^{4xy}$.

23. (10 points) Find $\frac{dy}{dx}$. **Your answer must be a function in x only.**

$$y = (\cos(5x))^{\sqrt{x}}$$

24. (12 points) Water is pumped into an inverted conical tank at a constant rate of $4 \text{ m}^3/\text{min}$. The tank has a height of 9 m, and the diameter across the top is 6 m. How fast is the water level rising when the water is 3 m deep? (The volume of a cone is $V = \frac{1}{3}\pi r^2 h$).

DO NOT WRITE IN THIS TABLE.

| Question | Points Awarded | Points |
|----------|----------------|--------|
| 1-20 | | 60 |
| 21 | | 12 |
| 22 | | 6 |
| 23 | | 10 |
| 24 | | 12 |
| | | 100 |