MATH 151, SPRING 2022
COMMON EXAM II - VERSION B

LAST NAME(print): ______________________ FIRST NAME(print): ______________________

INSTRUCTOR: ______________________

SECTION NUMBER: ____________

DIRECTIONS:

1. No calculator, cell phones, or other electronic devices may be used, and they must all be put away out of sight.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. **Be sure to *fill in your name, UIN, section number and version letter of the exam* on the ScanTron form.**

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________________

PART I: Multiple Choice. 3 points each

1. Determine where the function f below is NOT differentiable.

$$f(x) = \begin{cases} \sin x & x < 0 \\ -x^3 + x & 0 \leq x < 1 \\ x^2 - 1 & 1 \leq x \end{cases}$$

   (a) f is differentiable everywhere
   (b) $x = 0$ only
   (c) $x = 1$ only $\leftarrow$ correct
   (d) $x = 0$ and $x = 1$
   (e) f is not differentiable anywhere

2. Find the equation of the tangent line to the graph of $f(x) = \dfrac{2x}{3x+1}$ at $x = 1$.

   (a) $y - \frac{1}{2} = -\frac{1}{8}\Big(x - 1\Big)$
   (b) $y - \frac{1}{2} = \frac{1}{8}\Big(x - 1\Big)$ $\leftarrow$ correct
   (c) $y - \frac{1}{2} = \frac{7}{8}\Big(x - 1\Big)$
   (d) $y - \frac{1}{2} = -\frac{2}{3}\Big(x - 1\Big)$
   (e) $y - \frac{1}{2} = -\frac{7}{8}\Big(x - 1\Big)$

3. Find the equation of the tangent line to the graph of $x = 3t^2 - 2t + 1$, $y = t^2 + 1$ at the point where $t = 1$.

   (a) $y = \frac{1}{2}x - 1$
   (b) $y = 2x - 2$
   (c) $y = \frac{1}{2}x + 1$ $\leftarrow$ correct
   (d) $y = x$
   (e) None of these

Suppose f and g are differentiable functions which satisfy the following condition. Use the following table for Problems 4 and 5.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	-2	1	1	1
1	-1	3	3	0

4. Let $u(x) = \dfrac{f(x)}{g(x)}$. Find $u'(1)$.

   (a) -1

   (b) 3

   (c) 0

   (d) 1 $\leftarrow$ correct

   (e) 2

5. Let $v(x) = g(f(x))$. Find $v'(1)$.

   (a) 0

   (b) -2

   (c) -1

   (d) 3 $\leftarrow$ correct

   (e) 5

6. Find the slope of the tangent line to the curve $x^2 - y^2 = 16$ at the point $(-5, 3)$.

   (a) $\dfrac{5}{3}$

   (b) $-\dfrac{5}{4}$

   (c) $\dfrac{4}{5}$

   (d) $\dfrac{3}{4}$

   (e) $-\dfrac{5}{3}$ $\leftarrow$ correct

7. The length of a rectangle is increasing at a rate of 5 cm/sec and its width is decreasing at a rate of 4 cm/sec. When the length is 20 cm and the width is 10 cm, what is the rate of change of the area of the rectangle?

   (a) -30 cm^2/sec $\leftarrow$ correct
   (b) 30 cm^2/sec
   (c) 60 cm^2/sec
   (d) -60 cm^2/sec
   (e) 130 cm^2/sec

8. Find all point(s) on the curve parametrized by $x = t^2 - 2t - 3$, $y = t^3 - 3t^2$ where the tangent line is horizontal.

   (a) $(-4, -2)$
   (b) $(-3, -4)$ and $(-3, 0)$ $\leftarrow$correct
   (c) $(0, 0)$ and $(-3, 0)$
   (d) $(0, 0)$ and $(0, -4)$
   (e) None of these

9. Find all point(s) on the curve parametrized by $x = t^2 - 2t - 3$, $y = t^3 - 3t^2$ where the tangent line is vertical.

   (a) $(0, 0)$ and $(-3, 0)$
   (b) $(-3, -4)$ and $(-3, 0)$
   (c) $(-4, -2)$ $\leftarrow$correct
   (d) $(0, 0)$ and $(0, -4)$
   (e) None of these

10. Find the derivative of the function $f(x) = \arcsin(e^{5x})$.

(a) $\dfrac{5e^{5x}}{\sqrt{1-e^{10x}}}$ $\leftarrow$ correct

(b) $\dfrac{5e^{5x}}{1+e^{10x}}$

(c) $\dfrac{1}{\sqrt{1-e^{10x}}}$

(d) $-\dfrac{5e^{5x}}{\sqrt{1-e^{10x}}}$

(e) $-\dfrac{5e^{5x}}{1+e^{10x}}$

11. Find $f'(0)$ for $f(x) = 2^{(3^x)}$.

(a) $\ln 2 \cdot \ln 3$

(b) $24 \ln 2 \cdot \ln 3$

(c) $3 \ln 2 \cdot \ln 3$

(d) $8 \ln 2 \cdot \ln 3$

(e) $2 \ln 2 \cdot \ln 3$ $\leftarrow$ correct

12. Use the linearization of $f(x) = \sqrt{x}$ at $x = 9$ to estimate the value of $\sqrt{9.1}$.

(a) $\dfrac{179}{60}$

(b) $\dfrac{19}{6}$

(c) $\dfrac{17}{6}$

(d) $\dfrac{181}{60}$ $\leftarrow$ correct

(e) $\dfrac{31}{30}$

13. The radius of a circle was measured to be 5 ft with a maximum possible error of 0.2 ft. Use differentials to estimate the maximum possible error in the calculated area of the circle.

(a) $\frac{2\pi}{25}$

(b) 10π

(c) 0.9π

(d) 0.09π

(e) 2π $\leftarrow$ correct

14. Find the 2021^{st} derivative of $f(x) = 2\cos(2x)$

(a) $-2^{2021}\sin(2x)$

(b) $-2^{2022}\sin(2x)$ $\leftarrow$ correct

(c) $2^{2022}\sin(2x)$

(d) $2^{2022}\cos(2x)$

(e) $-2^{2021}\cos(2x)$

15. The position of a particle is given by the vector function $\mathbf{r}(t) = \langle te^t, t^3\rangle$. Find the acceleration vector of the particle at time $t = 1$.

(a) $\langle e, 6\rangle$

(b) $\langle 2e, 6\rangle$

(c) $\langle 2e, 3\rangle$

(d) $\langle 3e, 6\rangle$ $\leftarrow$ correct

(e) $\langle e, 3\rangle$

16. Find $f'(x)$ for $f(x) = \ln\left(\dfrac{\sqrt{x^6+1}}{\sec^{10} x}\right)$ [HINT: Use properties of logarithms.]

(a) $\dfrac{1}{12x^5} - \dfrac{10}{\sec x \tan x}$

(b) $\dfrac{1}{12x^5} - \dfrac{1}{10 \sec x \tan x}$

(c) $\dfrac{3x^5}{x^6+1} - \dfrac{10 \tan}{\sec x}$

(d) $\dfrac{3x^5}{x^6+1} - \tan x$

(e) $\dfrac{3x^5}{x^6+1} - 10 \tan x$ ← correct

17. A ball is tossed in the air, and the height of the ball at time t seconds is given by $h(t) = 25 + 10t - t^2$, where $h(t)$ is measured in feet from the ground. Find the maximum height H of the ball and the time T when it hits the maximum height.

(a) $H = 25$, $T = 5$

(b) $H = 25$, $T = 10$

(c) $H = 50$, $T = 10$

(d) $H = 50$, $T = 5$ ← correct

(e) $H = 30$, $T = 5$

18. A particle moves according to the equation of motion $s(t) = t^2 - 2t + 3$ where $s(t)$ is measured in meter and t is measured in seconds. Find the total distance traveled in the first 3 seconds.

(a) 6 m

(b) 5 m ← correct

(c) 4 m

(d) 3 m

(e) 2 m

19. A bacteria culture starts with 2 million bacteria, and the population triples every 30 minutes. Find the number of bacteria after 90 minutes.

   (a) 16 million
   (b) 18 million
   (c) 30 million
   (d) 54 million $\leftarrow$ correct
   (e) 72 million

20. Find $h''(1)$ if $h(x) = e^{-x^2}$

   (a) $\frac{4}{e}$
   (b) $-\frac{2}{e}$
   (c) $\frac{2}{e}$ $\leftarrow$ correct
   (d) $\frac{1}{e}$
   (e) $-\frac{4}{e}$

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

21. (12 points) Find $\frac{dy}{dx}$ for the following functions. **Do not simplify after taking the derivative**.

(a) $y = \sec^3\left(4x^5 + \pi^2\right)$

Sol. Apply the Power Rule and Chain Rule to have

$$\begin{aligned}\frac{dy}{dx} =& 3\sec^2\left(4x^5+\pi^2\right)\Big(\sec\left(4x^5+\pi^2\right)\Big)' \\ =& 3\sec^2\left(4x^5+\pi^2\right)\sec\left(4x^5+\pi^2\right)\tan\left(4x^5+\pi^2\right)\Big(4x^5+\pi^2\Big)' \\ =& 3\sec^2\left(4x^5+\pi^2\right)\sec\left(4x^5+\pi^2\right)\tan\left(4x^5+\pi^2\right)\Big(20x^4\Big)\end{aligned}$$

(b) $y = \dfrac{3^x \tan x}{x^4}$

Sol. Apply the Quotient Rule and Chain Rule to have

$$\begin{aligned}\frac{dy}{dx} =& \frac{\left(3^x\tan x\right)'(x^4) - 3^x\tan x\left(x^4\right)'}{(x^4)^2} \\ =& \frac{(3^x\ln 3\tan x + 3^x\sec^2 x)(x^4) - 3^x\tan x\left(4x^3\right)}{x^8}\end{aligned}$$

Alternatively, one can apply Logarithmic Differentiation.

$$\begin{aligned}\ln y =& \ln\left(\frac{3^x\tan x}{x^4}\right) = x\ln 3 + \ln\tan x - 4\ln x \\ \Rightarrow\quad \frac{y'}{y} =& \ln 3 + \frac{\sec^2 x}{\tan x} - \frac{4}{x} \\ \Rightarrow\quad y' =& \left(\ln 3 + \frac{\sec^2 x}{\tan x} - \frac{4}{x}\right)\frac{3^x\tan x}{x^4}\end{aligned}$$

22. (6 points) Find $\dfrac{dy}{dx}$ for the equation $y = e^{4xy}$.

Sol. Apply Implicit Differentiation to have

$$\begin{aligned} y' =& e^{4xy}\Big(4xy\Big)' = e^{4xy}\Big(4y + 4xy'\Big) = e^{4xy}4y + e^{4xy}4xy' \\ \Rightarrow&\quad y' - e^{4xy}4xy' = e^{4xy}4y \\ \Rightarrow&\quad \Big(1 - e^{4xy}4x\Big)y' = e^{4xy}4y \end{aligned}$$

So we have

$$\frac{dy}{dx} = \frac{4e^{4xy}y}{1 - 4e^{4xy}x}$$

23. (10 points) Find $\dfrac{dy}{dx}$. **Your answer must be a function in x only.**

$y = \Big(\cos(5x)\Big)^{\sqrt{x}}$

Sol. Take ln of $y = \big(\cos 5x\big)^{\sqrt{x}}$ to have

$$\ln y = \ln\big(\cos 5x\big)^{\sqrt{x}} = \sqrt{x}\cdot\ln(\cos 5x)$$

Apply Implicit Differentiation to have

$$\begin{aligned} \frac{y'}{y} =& \big(\sqrt{x}\big)'\cdot\ln(\cos 5x) + \sqrt{x}\cdot\big(\ln(\cos 5x)\big)' \\ =& \big(\frac{1}{2}x^{-1/2}\big)\ln(\cos 5x) + \sqrt{x}\cdot\frac{1}{\cos 5x}(-\sin 5x)5 \end{aligned}$$

Thus we have

$$y' = \Big(\frac{1}{2\sqrt{x}}\ln(\cos 5x) + \sqrt{x}\cdot\frac{-5\sin 5x}{\cos 5x}\Big)\big(\cos 5x\big)^{\sqrt{x}}$$

Acceptible answers include

$$y = \Big(\frac{1}{2\sqrt{x}}\ln(\cos 5x) - 5\sqrt{x}\tan 5x\Big)\big(\cos 5x\big)^{\sqrt{x}}$$

24. (12 points) Water is pumped into an inverted conical tank at a constant rate of 4 $\mathrm{m}^3/\mathrm{min}$. The tank has a height of 9 m, and the diameter across the top is 6 m. How fast is the water level rising when the water is 3 m deep? (The volume of a cone is $V = \frac{1}{3}\pi r^2 h$).

Sol. From the condition of height $h = h(t)$ and the radius $r = r(t)$, we have

$$h : 2r = 9 : 6 \quad \Rightarrow \quad r(t) = \frac{h(t)}{3}$$

The volume can be expressed as

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h = \frac{\pi h^3}{27}$$

The rate of change in volume $V = V(h)$ is

$$\frac{dV}{dt} = \frac{\pi}{27}\left(h^3(t)\right)' = \frac{\pi}{27}\,3h^2\,h'(t) = \frac{\pi h^2}{9}\cdot\frac{dh}{dt}$$

The condition implies $h = 3$, $\frac{dV}{dt} = 4$. Thus we have

$$4 = \frac{\pi 3^2}{9}\cdot\frac{dh}{dt} \quad \Rightarrow \quad \frac{dh}{dt} = \frac{4}{\pi}\ (\mathrm{m/min}).$$

DO NOT WRITE IN THIS TABLE.

Question	Points Awarded	Points
1-20		60
21		12
22		6
23		10
24		12
		100