MATH 152, Fall 2022

COMMON EXAM II - VERSION A

LAST NAME(print): ______________________ FIRST NAME(print): ______________________

INSTRUCTOR: ______________________

UIN: ______________________

SECTION NUMBER: ____________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. **Be sure to *fill in your name, UIN, section number and version letter of the exam* on the *ScanTron form.***

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________________

Some integrals that may or may not be useful.

$$\int \sec^3 x\ dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$$

$$\int \csc^3 x\ dx = \frac{-1}{2}\csc x \cot x + \frac{1}{2}\ln|\csc x - \cot x| + C$$

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PART I: Multiple Choice. 3.5 points each

1. Let $\sum_{n=1}^{\infty} a_n$ be a series whose nth partial sum is $s_n = \dfrac{7n^2+5}{5n^2+2}$. The series

   (a) converges to $\dfrac{12}{7}$
   (b) converges to 2.5
   (c) diverges
   (d) converges to $\dfrac{7}{5}$
   (e) None of these.

2. After an appropriate substitution, the integral $\int x^2\sqrt{9-x^2}\,dx$ is equivalent to which of the following?

   (a) $9\int \cos^2\theta\ d\theta$
   (b) $81\int \sin^2\theta\cos^2\theta d\theta$
   (c) $27\int \sin^2\theta\cos\theta\ d\theta$
   (d) $81\int \sec^3\theta\tan^2\theta\ d\theta$
   (e) $27\int \sec^2\theta\tan\theta\ d\theta$

3. Compute $\displaystyle\int_0^4 \frac{x+2}{x^2+4}\ dx$.

   (a) $\dfrac{1}{2}(\ln 20-\ln 4)+\arctan(2)$
   (b) $\ln 6-\ln 2$
   (c) $\ln 20-\ln 4$
   (d) $\dfrac{1}{2}(\ln 20-\ln 4)+2\arctan(4)$
   (e) $\ln 20-\ln 4+2\arctan(4)$

4. Let $\sum_{n=1}^{\infty} a_n$ be a series whose nth partial sum is $s_n = \frac{n}{n+2}$. Find a_4.

(a) $a_4 = \frac{2}{3}$

(b) None of these.

(c) $a_4 = \frac{1}{21}$

(d) $a_4 = 1$

(e) $a_4 = \frac{1}{15}$

5. Compute $\int_{-1}^{\infty} \frac{1}{1+x^2}\, dx$.

(a) ∞

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$

(d) None of these.

(e) $\frac{3\pi}{4}$

6. Which sequence is both bounded and increasing?

(a) $a_n = \sin(2n\pi)$

(b) None of these.

(c) $a_n = e^{-n}$

(d) $a_n = 1 - \frac{2}{n}$

(e) $a_n = \ln n$

7. The sequence $a_n = \frac{(-1)^n n^2}{2n^2+5}$

(a) Diverges

(b) Converges to $\frac{1}{2}$

(c) None of these.

(d) Converges to 0

(e) Converges to $\frac{-1}{2}$

8. Which of the following is an appropriate substitution to use when solving the integral $\int \sqrt{16x^2 - 9}\, dx$?

(a) $x = \frac{3}{4} \sin\theta$
(b) $x = \frac{4}{3} \sec\theta$
(c) $x = \frac{4}{3} \sin\theta$
(d) $x = \frac{3}{4} \sec\theta$
(e) $x = \frac{3}{4} \tan\theta$

9. Which of the following statements is true regarding the improper integral $\int_1^\infty \frac{1}{e^x + \sqrt{x}}\, dx$?

(a) The integral converges because $\int_1^\infty \frac{1}{e^x + \sqrt{x}}\, dx < \int_1^\infty \frac{1}{\sqrt{x}}\, dx$ and $\int_1^\infty \frac{1}{\sqrt{x}}\, dx$ converges.
(b) The integral diverges because $\int_1^\infty \frac{1}{e^x + \sqrt{x}}\, dx > \int_1^\infty \frac{1}{e^x}\, dx$ and $\int_1^\infty \frac{1}{e^x}\, dx$ diverges.
(c) The integral diverges because $\int_1^\infty \frac{1}{e^x + \sqrt{x}}\, dx > \int_1^\infty \frac{1}{\sqrt{x}}\, dx$ and $\int_1^\infty \frac{1}{\sqrt{x}}\, dx$ diverges.
(d) The integral converges because $\int_1^\infty \frac{1}{e^x + \sqrt{x}}\, dx < \int_1^\infty \frac{1}{e^x}\, dx$ and $\int_1^\infty \frac{1}{e^x}\, dx$ converges.
(e) The integral converges to 0.

10. Compute the sum of the series $\sum_{n=1}^{\infty} \frac{(-4)^{n+1}}{5^n}$.

(a) This series diverges.
(b) $\frac{16}{9}$
(c) $\frac{-20}{9}$
(d) $\frac{-16}{9}$
(e) None of these.

11. Which of the following is a proper Partial Fraction Decomposition for the rational function

$$\frac{5x+1}{(x+3)(x^2+4x+3)(x^2+4)}$$

(a) $\dfrac{A}{x+3}+\dfrac{B}{(x+3)^2}+\dfrac{C}{x+1}+\dfrac{Dx+E}{x^2+4}$

(b) $\dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+4x+3}+\dfrac{Dx+E}{x^2+4}$

(c) $\dfrac{A}{x+3}+\dfrac{Bx+C}{(x+3)^2}+\dfrac{D}{x+1}+\dfrac{Ex+F}{x^2+4}$

(d) $\dfrac{A}{x+3}+\dfrac{B}{(x+3)^2}+\dfrac{C}{x+1}+\dfrac{D}{x+2}+\dfrac{D}{x-2}$

(e) None of these.

12. Assume that the sequence $\{a_n\}$ is decreasing and bounded below by 1, i.e. $a_n \geq 1$, for all positive n. Determine if the sequence is convergent or divergent.

$$a_1 = 4 \qquad \text{and} \qquad a_{n+1} = \frac{10}{7-a_n}$$

(a) Convergent to 2

(b) Convergent to 1

(c) Divergent

(d) Convergent to $\dfrac{10}{7}$

(e) Convergent to 5

13. Which of the following series diverges by the Test for Divergence?

(I) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{\pi n}{2n+1}\right)$ (II) $\displaystyle\sum_{n=1}^{\infty} \frac{4}{4+e^{-3n}}$ (III) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\arctan n}$

(a) (II) only

(b) (III) only

(c) (I), (II), and (III)

(d) (I) and (II) only

(e) (II) and (III) only

14. Which of these substitutions would be used to evaluate $\int x^2\sqrt{x^2+4x+13}\,dx$?

(a) $x+4=\sqrt{13}\sec\theta$
(b) $x+2=3\tan\theta$
(c) $x^2+4x=\sqrt{13}\tan\theta$
(d) none of these.
(e) $x+2=3\sec\theta$

15. Let $s=\sum_{n=1}^{\infty}\frac{1}{n^3}$ Using The Remainder Estimate for the Integral Test, determine the smallest value of n that ensures that $R_n=s-s_n\le\frac{1}{44}$.

(a) n = 5
(b) n = 7
(c) n = 8
(d) n = 6
(e) n = 4

16. The series $\sum_{i=1}^{\infty}\left(e^{1/i}-e^{1/(i+1)}\right)$

(a) converges to e
(b) converges to 0
(c) converges to $e-1$
(d) None of these.
(e) diverges

17. The improper integral $\int_1^e \frac{1}{x \ln x}\, dx$

(a) diverges to $-\infty$.
(b) converges to 1.
(c) diverges to ∞.
(d) converges to -1.
(e) converges to $\frac{1}{e} - 1$.

18. The sequence $a_n = \frac{n^2}{n+2} - \frac{n^2}{n+5}$

(a) Converges to 0
(b) None of these.
(c) Converges to 3
(d) Diverges
(e) Converges to 7

PART II WORK OUT

<u>Directions</u>: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

19. (6 points) Find a general formula, a_n, for the sequence. Assume the pattern continues, and begins with $n = 1$.

$$\left\{ \frac{-5}{4}, \frac{8}{9}, \frac{-11}{16}, \frac{14}{25}, \frac{-17}{36}, \ldots \right\}$$

20. (5 points) Determine whether the series converges or diverges. Fully support your conclusion.

$$\sum_{n=1}^{\infty} \frac{2^{3n}}{7^n}$$

21. (6 points) Determine whether the series converges or diverges. Fully support your conclusion.

$$\sum_{n=1}^{\infty} ne^{-n^2}$$

22. (10 points) Compute $\int \frac{1}{x^4\sqrt{x^2-4}}\,dx$. In your final answer, any trig or inverse trig expressions that can be rewritten algebraically must be.

23. (10 points) Compute $\int \frac{2x^2+5x-5}{(x+1)(x+3)^2}\, dx$

DO NOT WRITE IN THIS TABLE.

Question	Points Awarded	Points
1-18		63
19		6
20		5
21		6
22		10
23		10
		100