2018 Power Team
Texas A&M High School Mathematics Contest
October 2018

We denote by $S(n)$ the sum of the base 10 digits of a natural number n. For example, $S(2018) = 2 + 0 + 1 + 8 = 11$.

Problem 1. Find all positive integers n such that $S(5^n) = 2^n$.

Problem 2. Compute $S(S(S(2018^{2018})))$.

Problem 3. Find all positive integers n such that

$$n + S(n) + S(S(n)) + S(S(S(n))) = 2018.$$

Problem 4. Prove the following inequalities for all natural numbers m and n

a) $S(m + n) \leq S(m) + S(n)$;
b) $S(mn) \leq S(m)S(n)$.

Problem 5. Prove that for every natural number n we have

a) $S(n) \leq 8S(8n)$;
b) $S(n) \leq 5S(5^5 n)$.

Problem 6. Prove that if $1 \leq x \leq 10^n$, then $S(x(10^n - 1)) = 9n$.

Problem 7. Find $S(9 \cdot 99 \cdot 9999 \cdot \ldots \cdot \underbrace{99\ldots99}_{2^n})$, where each factor has twice as many digits as the previous one.

Problem 8. Prove that for every positive integer n there exists a positive integer x such that $x + S(x) = n$ or $x + S(x) = n + 1$.

Problem 9. Prove that there exist 50 pairwise distinct positive integers n for which the value $n + S(n)$ is the same.

Problem 10. Does there exist n such that $S(n) = 1000$ and $S(n^2) = 1000^2$?

Problem 11.

a) Does there exist n such that
   i) $S(n^2) = 2018$?
   ii) $S(n^2) = 2017$?
b) Describe all k for which there exists n such that $S(n^2) = k$.