Modeling the Effects of Cannabalistic Behavior in Zebra Mussel (Dreissena polymorpha) Populations

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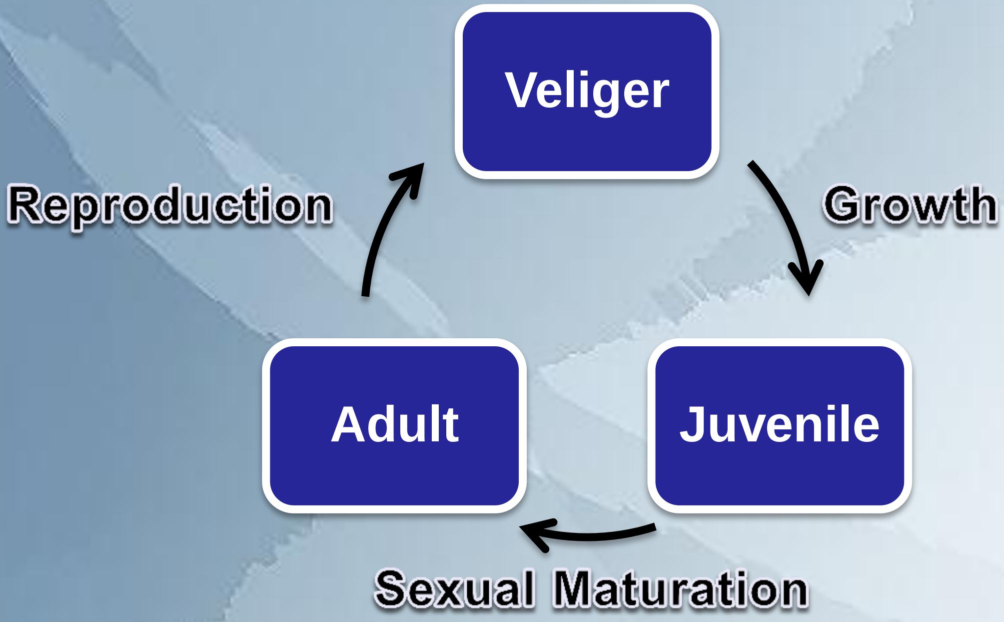
The Biology

- Small Freshwater European Bivalve Mollusk
- Three Life Stages:



- Longevity Ranges from 4 to 8 Years
- Filter Feeding → Cannibalistic Behavior

Picture from:
http://webhost.bridgew.edu/dpadgett/Course_page.htm

Picture from:
http://www.vpr.net/news_detail/86558/

The Issues

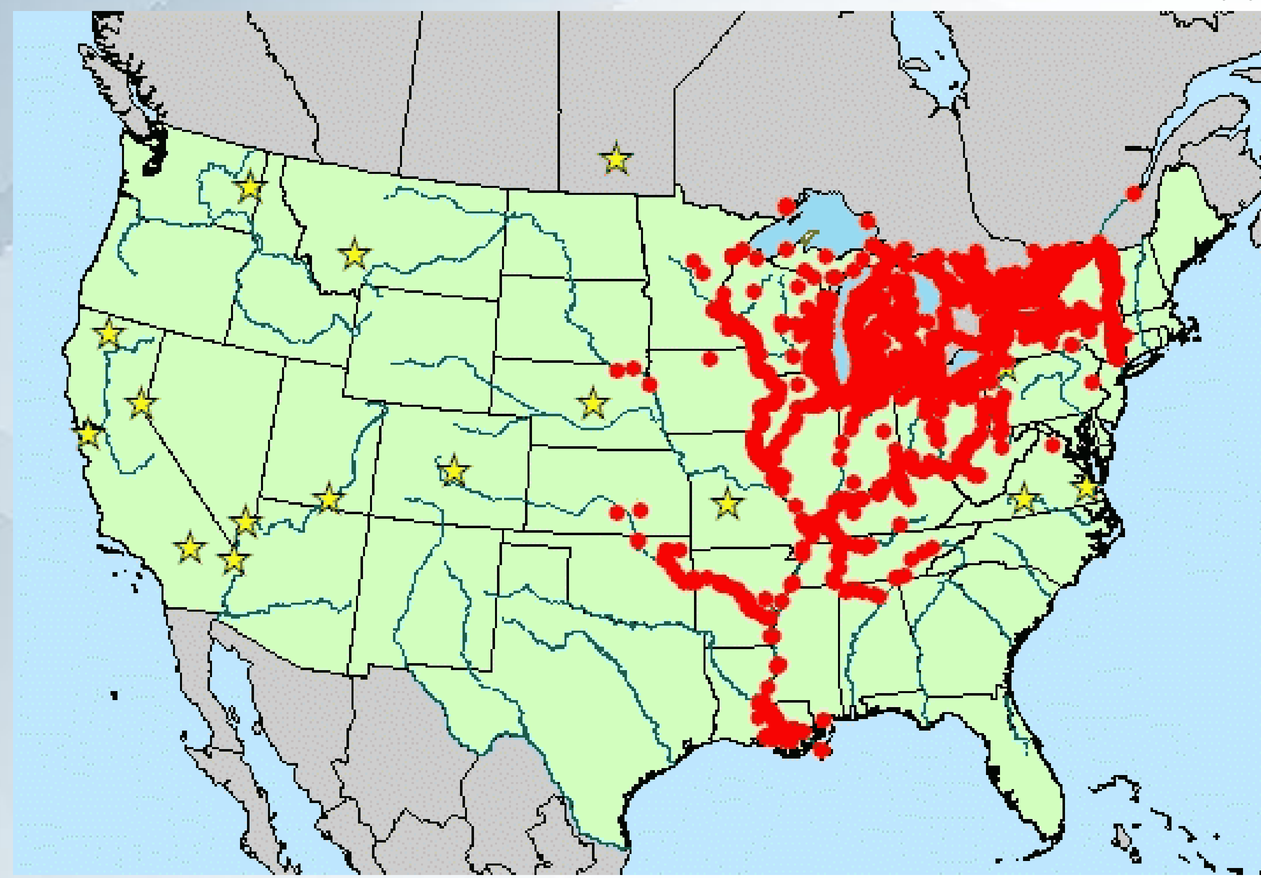
Picture from: http://www.invasivespeciesscotland.org.uk/invasive_non_native_species/zebra_mussel.asp

- Biofouling (unwanted accumulation of a certain population in an ecosystem)
  → increased competition for native species
- Attach to Solid Substrate
  → physical damage to artificial structures
  → clog intake valves of industrial facilities

The Issues

- Filter Feeding
  → removes large abundances of phytoplankton
  → toxin accumulation in predatory species

- In the United States
  → introduced in 1988 into Lake St. Clair
  → spread to all of the Great Lakes by 1990



Picture(s) from:
http://fl.biology.usgs.gov/Nonindigenous_Species/ZM_Progression/zm_progression.html

The Deterministic Model

Cannibalism

Reproduction

$$n_1(t+1) = \sigma_0\, exp\left[-\beta N(t)\right] \left[\frac{f_2 n_2(t)}{2} + \frac{f_3 n_3(t)}{2} + \frac{f_4 n_4(t)}{2}\right]$$

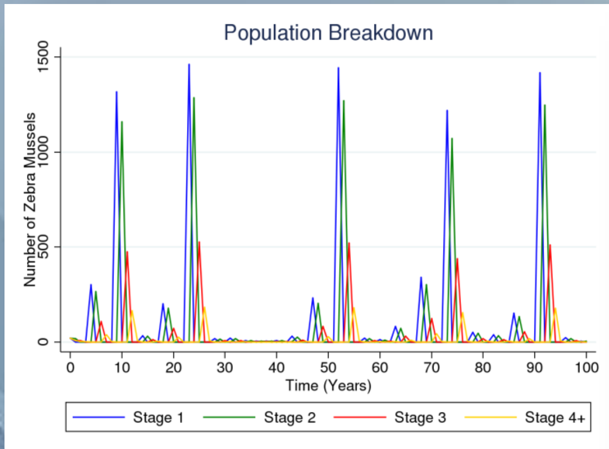
$$n_2(t+1) = \sigma_1 n_1(t)$$

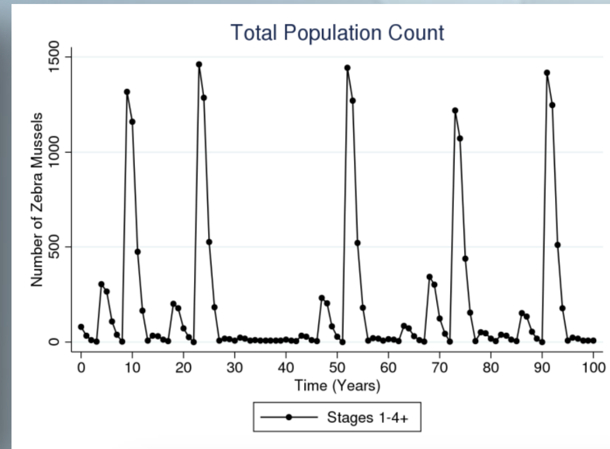
$$n_3(t+1) = \sigma_2 n_2(t)$$

$$n_4(t+1) = \sigma_3 n_3(t) + \sigma_4 n_4(t)$$

Parameter	Value	Description
σ_0	0.01	combined rate of veliger survival and birth
σ_1	0.88	survival rate of Stage 1 mussels to Stage 2
σ_2	0.41	survival rate of Stage 2 mussels to Stage 3
σ_3	0.35	survival rate of Stage 3 mussels to Stage 4+
σ_4	0.04	retention rate of Stage 4+ mussels
f_2	0.24×10^6	fecundity of Stage 2 female mussels
f_3	0.465×10^6	fecundity of Stage 3 female mussels
f_4	0.795×10^6	fecundity of Stage 4 female mussels
β	1.0	filtration rate of adult mussels

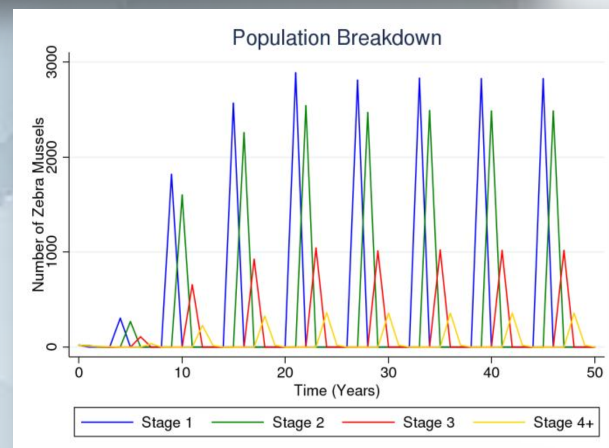
The Population Behaviors

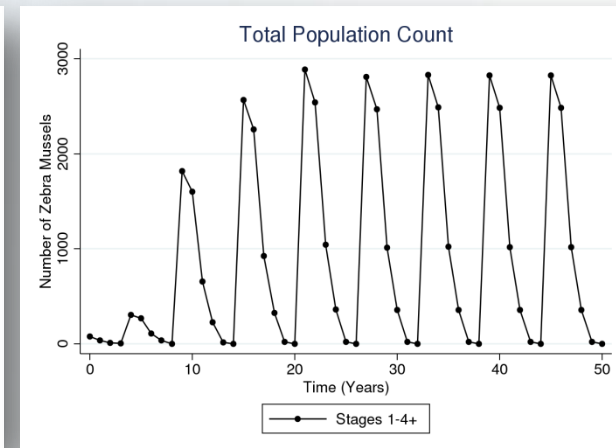


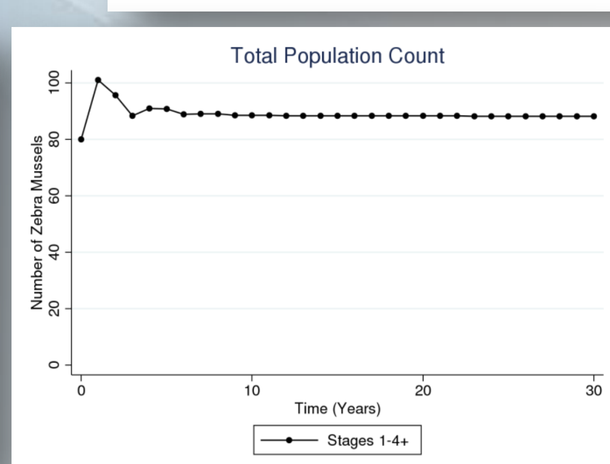


Chaotic Pattern

Cyclic Pattern



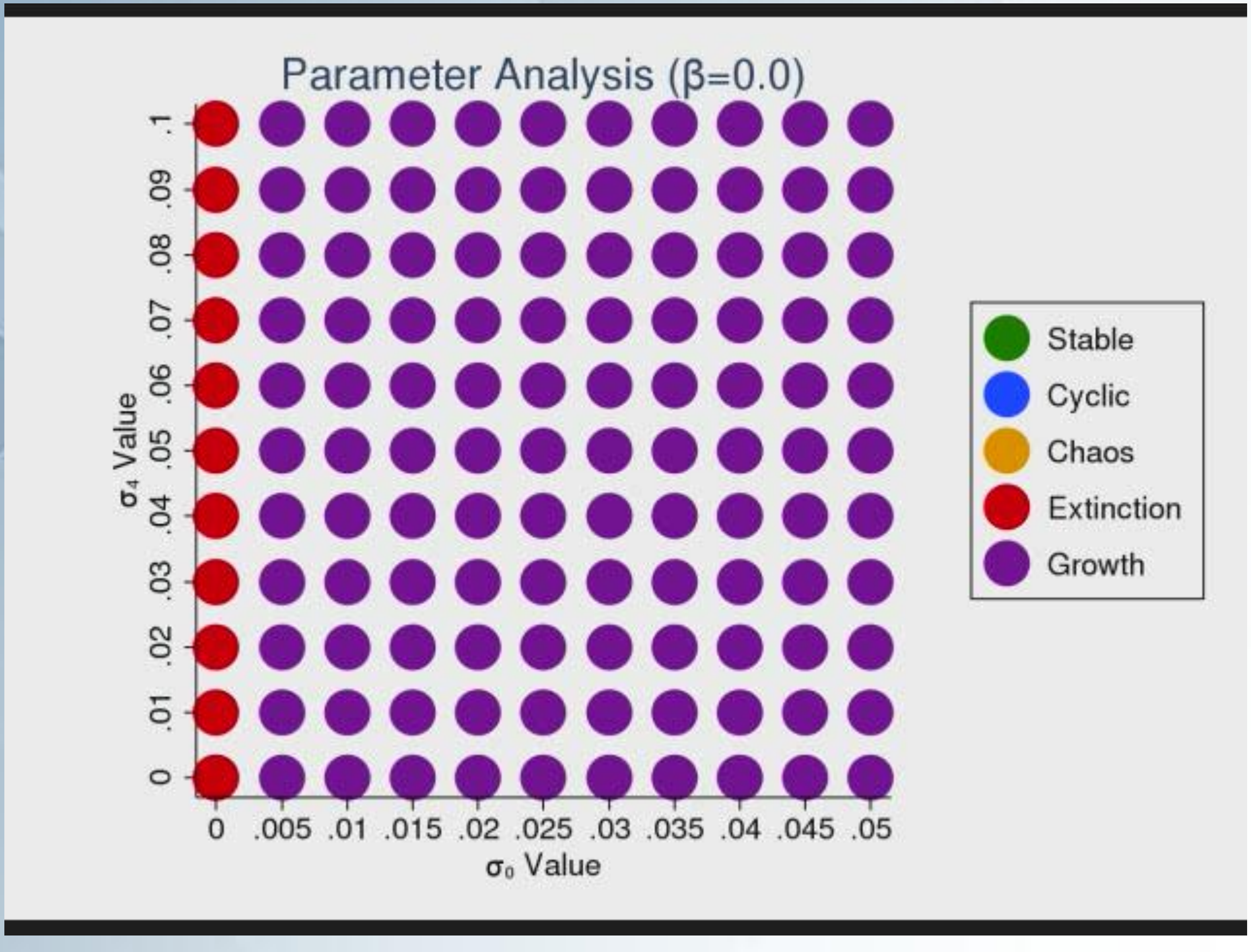




Stable Pattern

The Multidimensional Parameter Analysis

Parameter	Minimum	Step Size	Maximum
σ_0	0.0	0.005	0.05
σ_4	0.0	0.01	0.1
β	0.0	0.1	1.0



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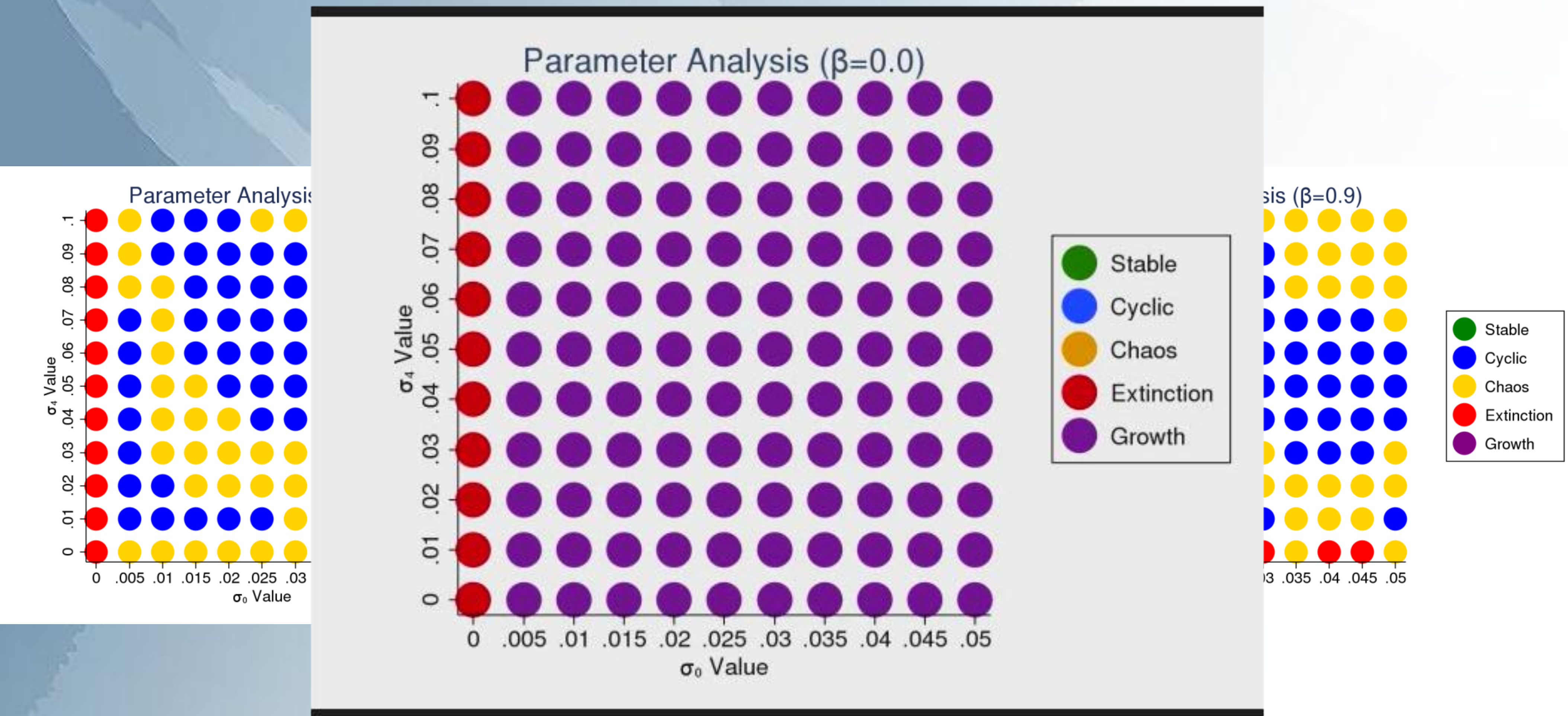
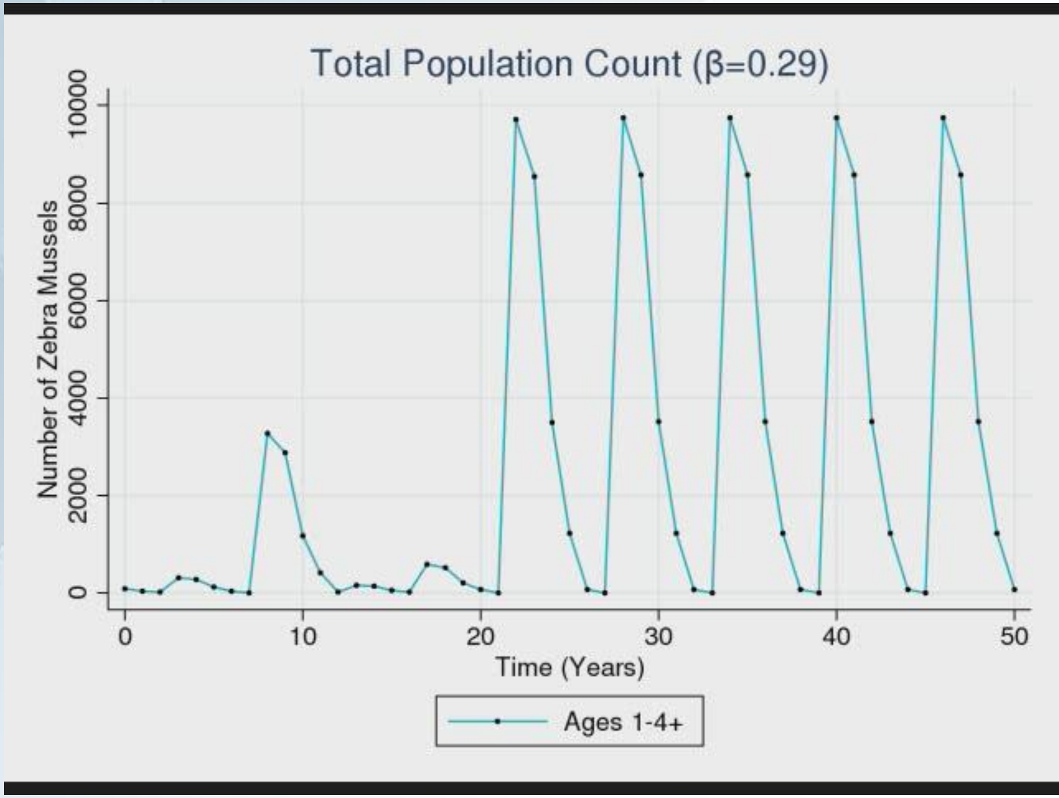
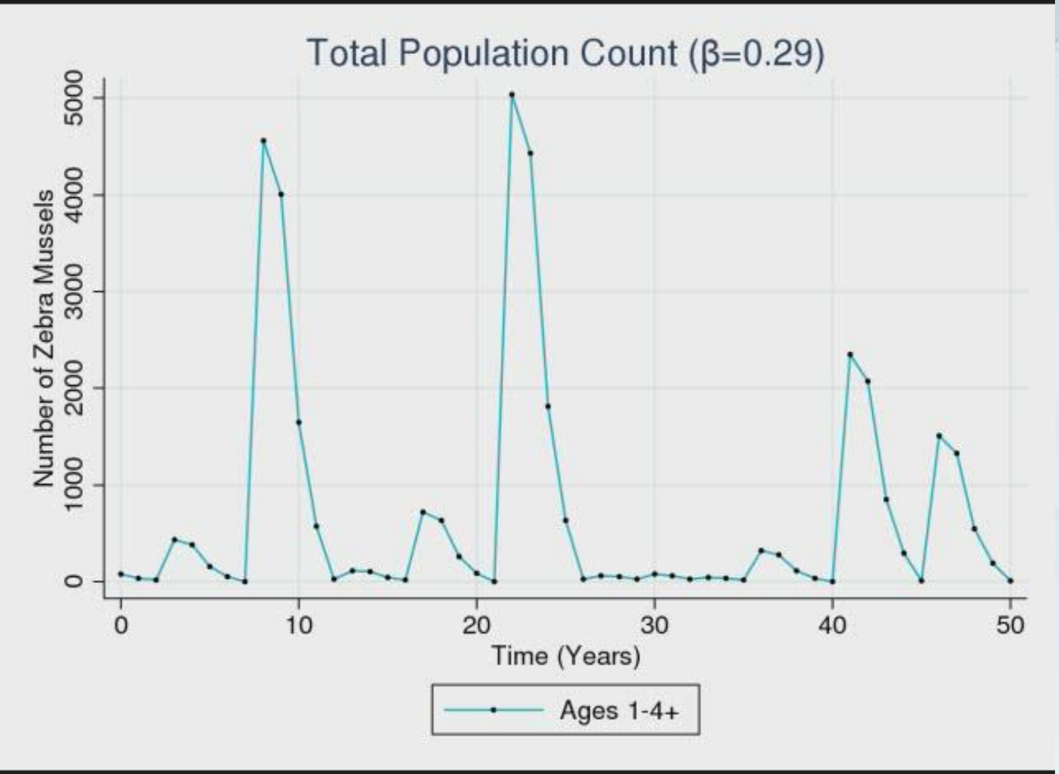


The β Parameter Analysis

Chaotic Pattern

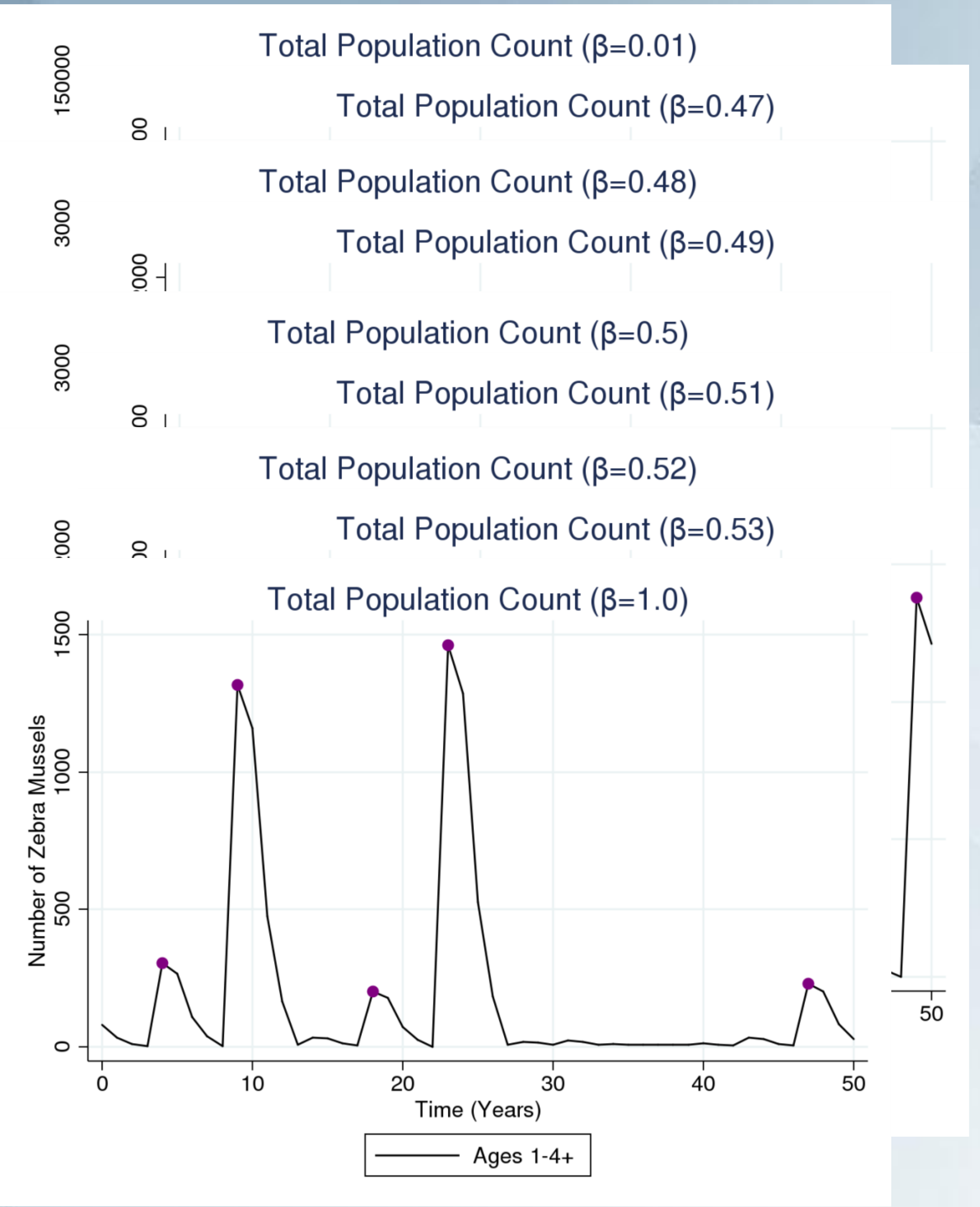




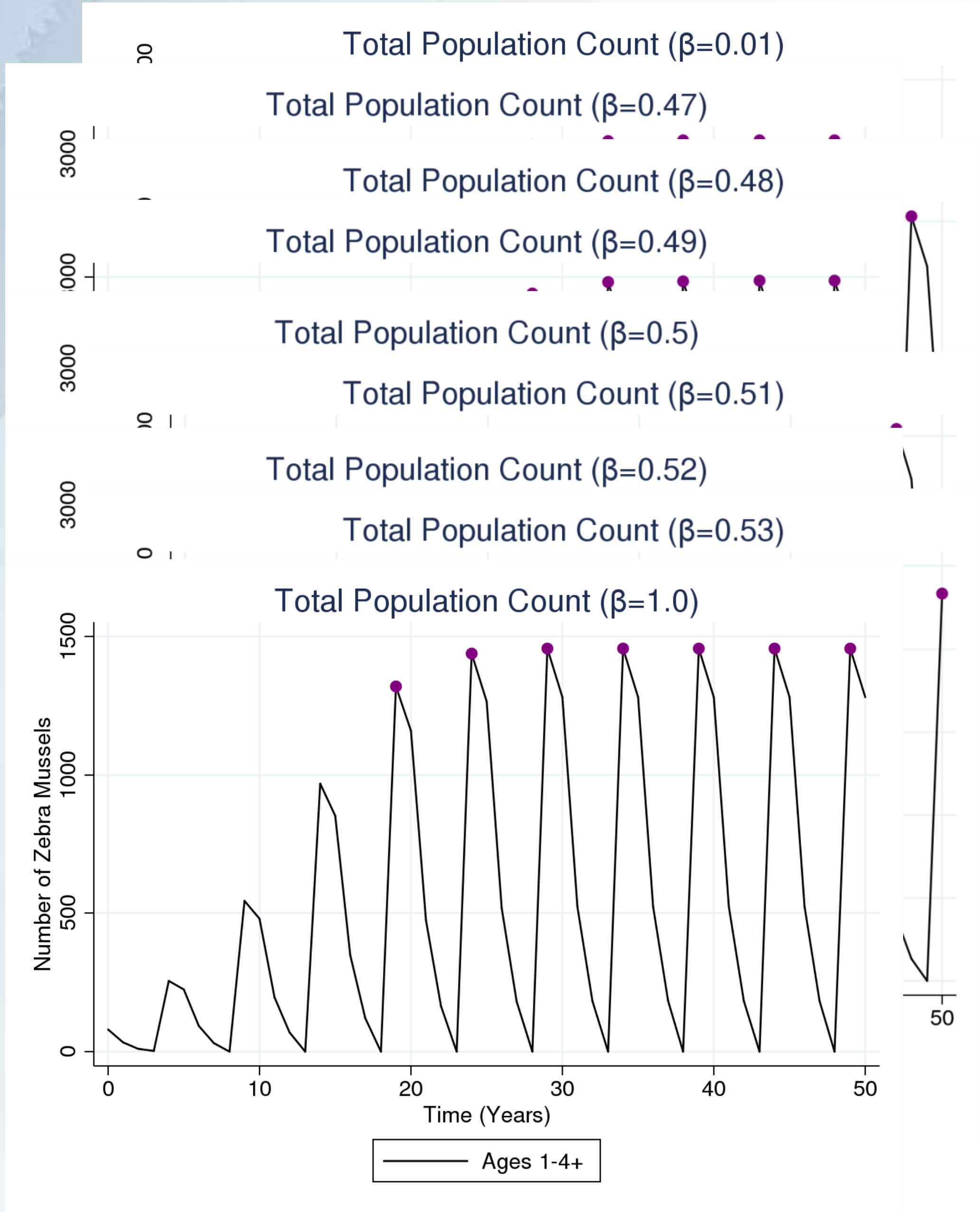
Cyclic Pattern

The β Parameter Analysis

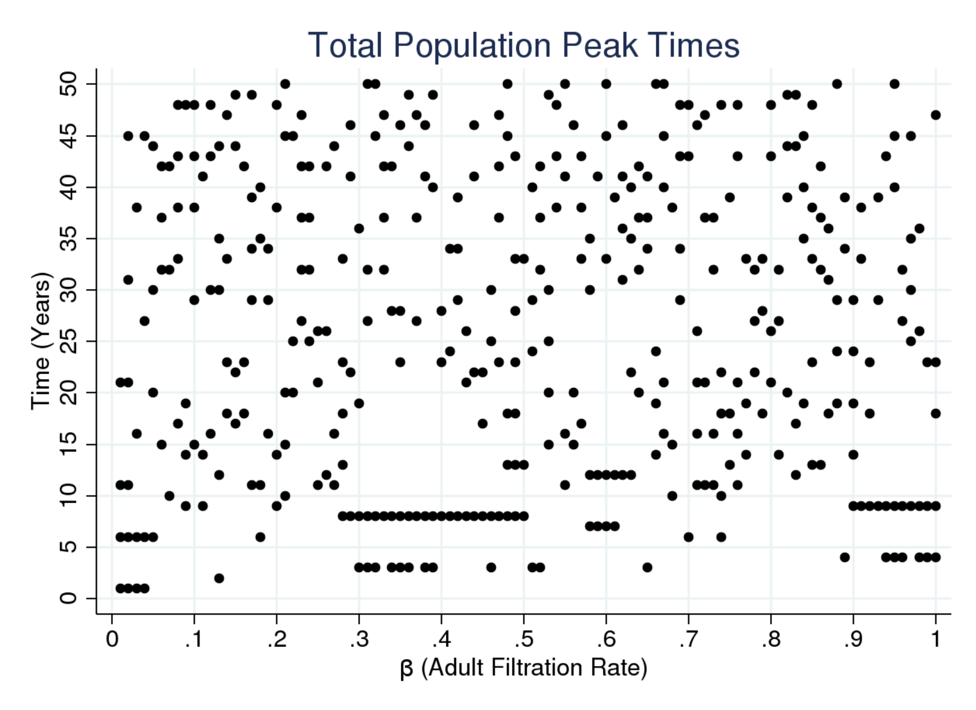
Chaotic Pattern



Cyclic Pattern

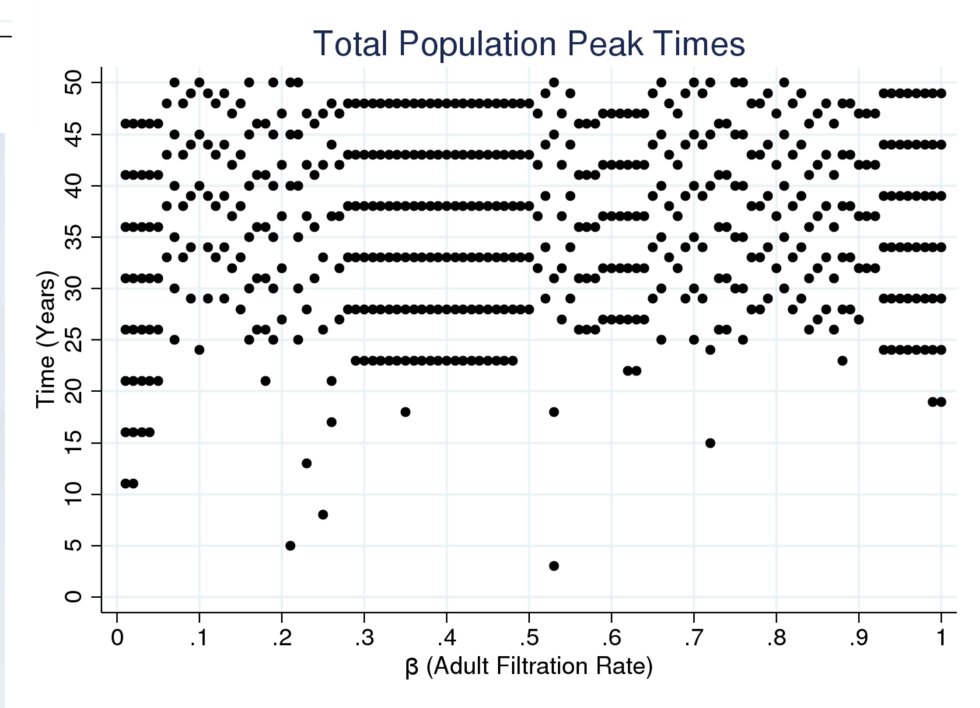


The β Parameter Analysis

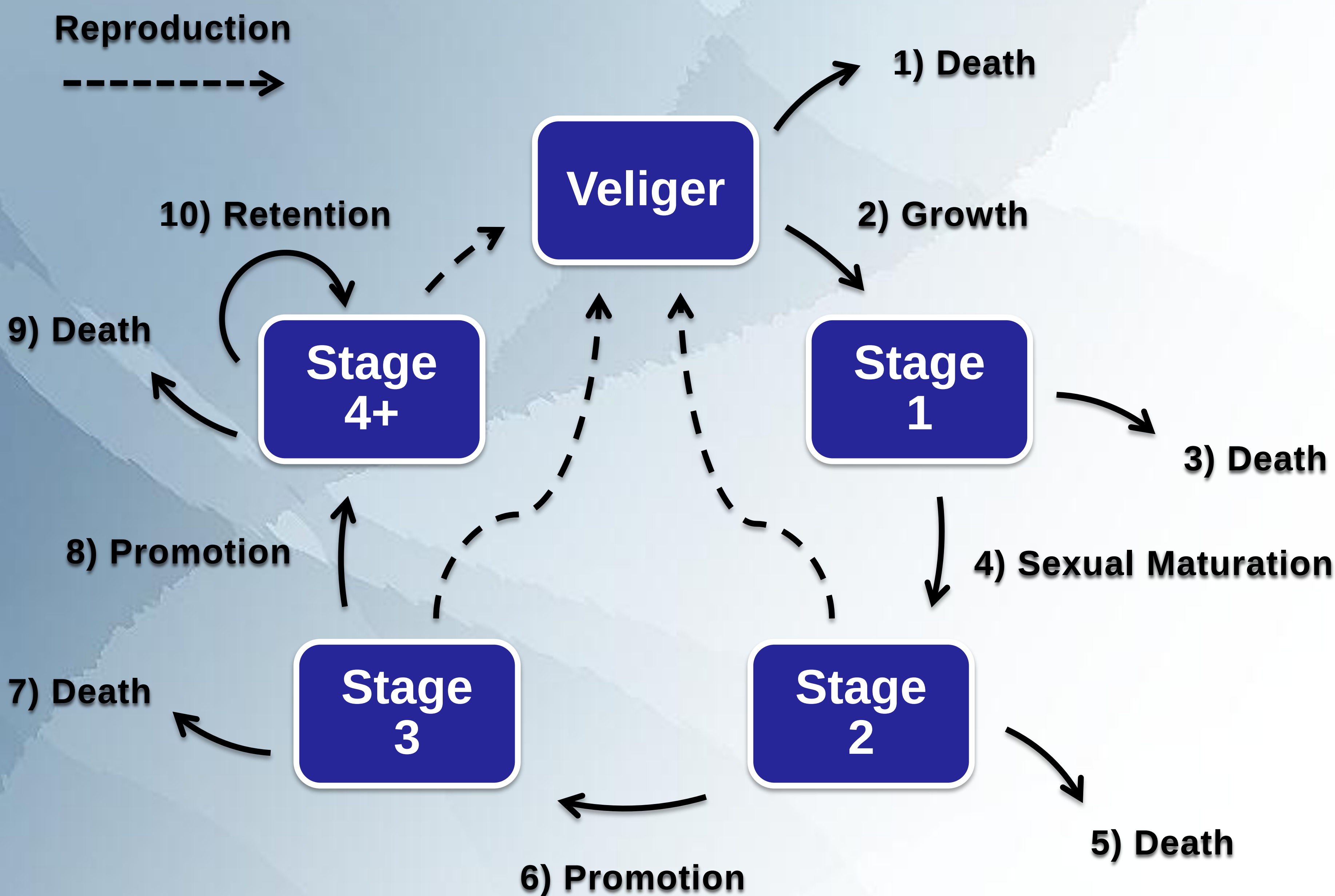


Chaotic Pattern

Cyclic Pattern



The Stochastic Model



The Stochastic Model

- Time and Event Generation Algorithm
  1) Start at $t_0 = 0$
  2) Generate a random number (θ_1) such that $0 \leq \theta_1 \leq 1$
  3) Calculate Δt based on an exponential distribution
  4) $t_{n+1} = t_n + \Delta t$
  5) Assign probabilities to each possible event

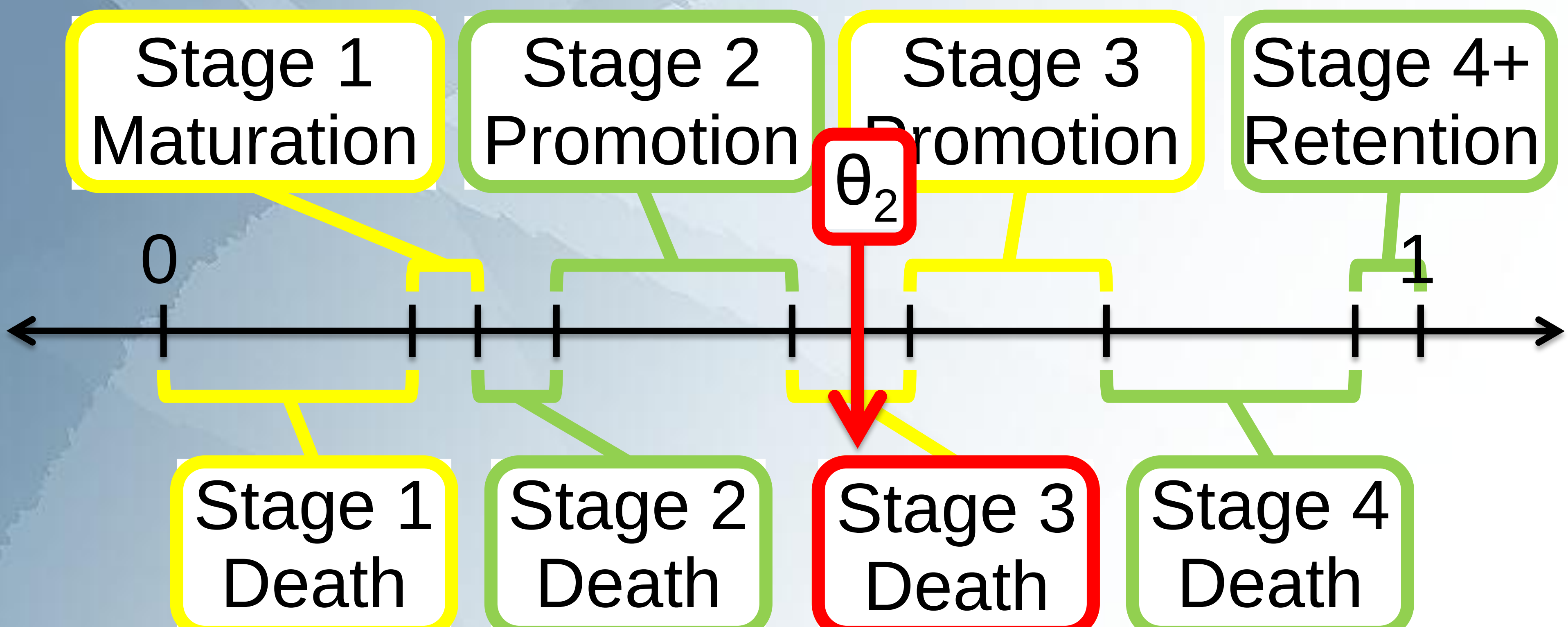
Event	Stage 1 Death	Stage 1 Maturation	Stage 2 Death	Stage 2 Promotion
Probability	$1 - \sigma_1$	σ_1	$1 - \sigma_2$	σ_2

Stage 3 Death	Stage 3 Promotion	Stage 4+ Death	Retention of Stage 4+
$1 - \sigma_3$	σ_3	$1 - \sigma_4$	σ_4

  6) Scale the probabilities down to add up to 1

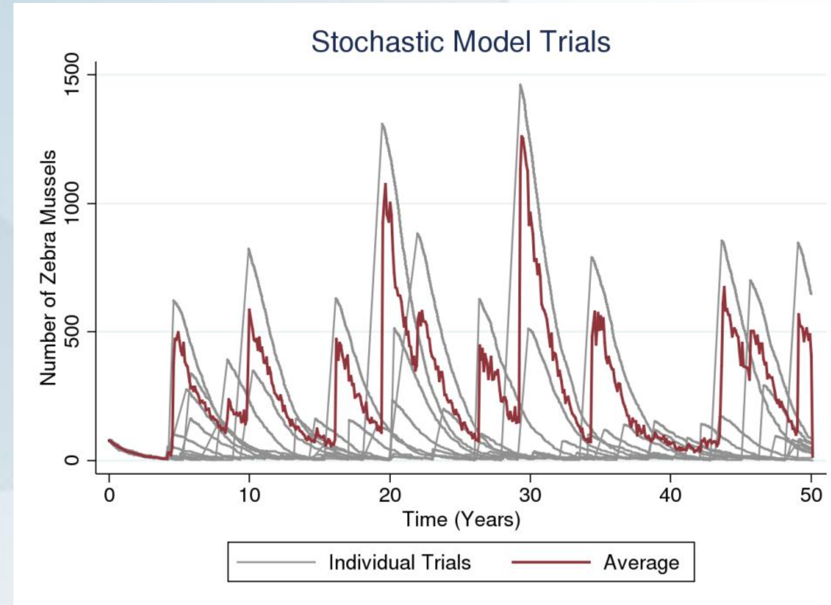
The Stochastic Model

- Time and Event Generation Algorithm
  7) Lay out on a theoretical number line
  8) Generate a random number (θ_2) such that $0 \leq \theta_2 \leq 1$
  9) Iterate steps until a maximum value of t is obtained

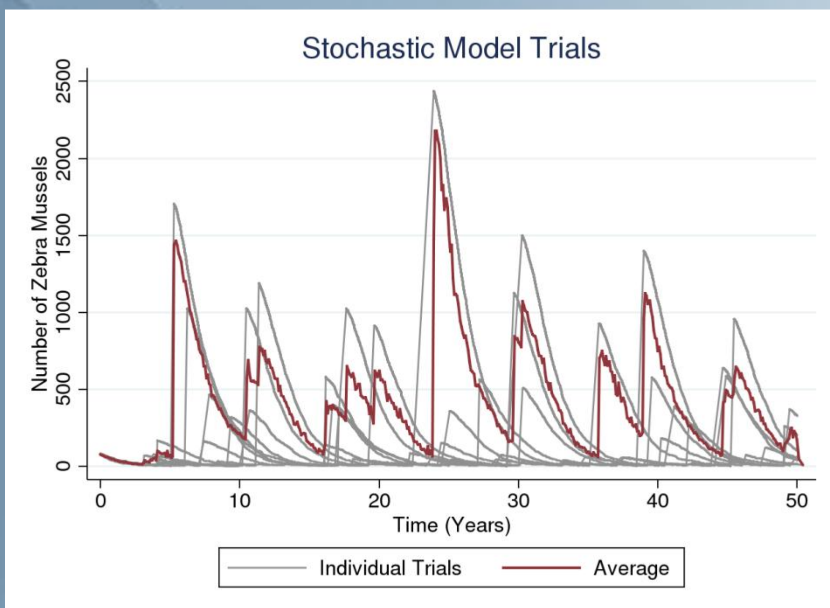


The Stochastic Graphs

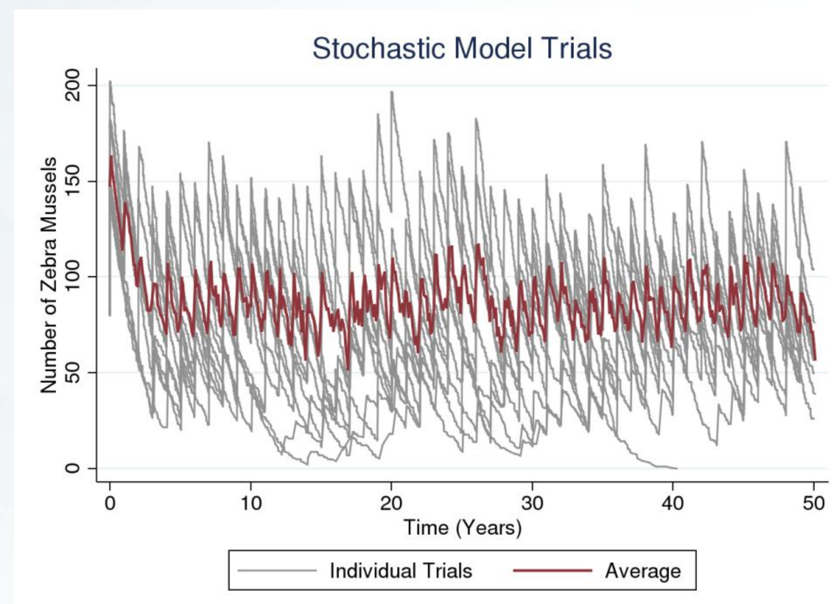
Chaotic Pattern



Cyclic Pattern



Stable Pattern



The References

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[1] The curse of the water hyacinth. *Economist*, 346(8050):68 – 69, 1998.

[2] D. Annoni, I. Bianchi, A. Girod, and M. Mariani. Inserimento di dreissena polymorpha (pallas)(mollusca bivalvia) nelle malacocenosi costiere del lago di garda (nord italia). *Quaderni della Civica Stazione Idrobiologica di Milano*, 6:5–84, 1978.

[3] P. W. Bartlett. Colorado beetles reported in england, wales and scotland, 1975. *Plant Pathology*, 25(1):44 – 47, 1976.

[4] Helen Buttery. In praise of a pest. *Maclean's*, 114(41):61, 2001.

[5] Renato Casagrandi, Lorenzo Mari, and Marino Gatto. Zebra mussel (dreissena polymorpha) effects on sediment, other zoobenthos, and the diet and growth of adult yellow perch (perca flavescens) in pond enclosures. *Canadian Journal of Fisheries and Aquatic Sciences*, 54(8):1903–1915, August 1997.

[6] Renato Casagrandi, Lorenzo Mari, and Marino Gatto. Modelling the local dynamics of the zebra mussel (dreissena polymorpha). *Freshwater Biology*, 52(7):1223–1238, July 2007.

[7] Daniel T. Gillespie. Exact stochastic simulation of coupled chemical reactions. *The Journal of Physical Chemistry*, 81:2340–2361, May 1977.

[8] Alexander Y. Karatayev, Lyubov E. Burlakova, and Dianna K. Padilla. Growth rate and longevity of dreissena polymorpha (pallas): A review and recommendations for future study. *Journal of Shellfish Research*, 25:23–32, April 2006.

[9] StataCorp LP. Stata statistical software: Release 11, 2009.

[10] Gerald L. Mackie and Don W. Schloesser. Comparative biology of zebra mussels in europe and north america: An overview. *American Zoologist*, 36(3):244–258, 1996.

[11] Lorenzo Mari, Renato Casagrandi, Maria Teresa Pisani, Emiliano Pucci, and Marino Gatto. When will the zebra mussel reach florence? a model for the spread of ¡i¿dreissena polymorpha¡/i¿ in the arno water system (italy). 2(4), 2009.

[12] Robert F. McMahon. The physiological ecology of the zebra mussel, dreissena polymorpha, in north america and europe. *American Zoologist*, 36(3):339–363, June 1996.

[13] Kristen M. Nelson, Carl R. Ruetz, and Donald G. Uzarski. Colonisation by dreissena of great lakes coastal ecosystems: how suitable are wetlands?. *Freshwater Biology*, 54(11):2290 – 2299, 2009.

[14] Charles R. O'Neill Jr. Economic impact of zebra mussels - results from the 1995 national zebra mussel information clearinghouse study. *Great Lakes Research Review*, 3(1), April 1997.

[15] Jeffrey L. Ram, Peter P. Fong, and David W. Garton. Physiological aspects of zebra mussel reproduction: Maturation, spawning, and fertilization. *American Zoologist*, 36(3):326–338, June 1996.

[16] Jeffrey L. Ram and Robert F. McMahon. Introduction: The biology, ecology, and physiology of zebra mussels. *American Zoologist*, 36(3):239–243, June 1996.

[17] Herbert Schildt. *Teach Yourself C*. Osborne McGraw-Hill, Berkeley, CA, 1990.

[18] Rowland H. Taylor and Bruce W. Thomas. Rats eradicated from rugged breaksea island (170 ha), fiordland, new zealand. *Biological Conservation*, 65(3):191 – 198, 1993.

[19] Levente Timar and Daniel J. Phaneuf. Modeling the human-induced spread of an aquatic invasive: The case of the zebra mussel. *Ecological Economics*, 68(12):3060 – 3071, 2009.

The Intrinsic Growth Rate

$$n_1(t+1) = \sigma_0 exp[-\beta N(t)] \left[\frac{f_2 n_2(t)}{2} + \frac{f_3 n_3(t)}{2} + \frac{f_4 n_4(t)}{2} \right]$$

$$N(t+1) = \sigma_0 exp[-\beta N(t)] \left[\frac{f_2 n_2(t)}{2} + \frac{f_3 n_3(t)}{2} + \frac{f_4 n_4(t)}{2} \right] + \sigma_1 n_1(t) + \sigma_2 n_2(t) + \sigma_3 n_3(t) + \sigma_4 n_4(t)$$

$$n_4(t+1) = \sigma_3 n_3(t) + \sigma_4 n_4(t)$$

$$\sigma_{min} N(t) \leq \sigma_1 n_1(t) + \sigma_2 n_2(t) + \sigma_3 n_3(t) + \sigma_4 n_4(t) \leq \sigma_{max} N(t)$$

The Intrinsic Growth Rate

$$N(t+1) = \sigma_0 exp[-\beta N(t)] \left[\frac{f_2 n_2(t)}{2} + \frac{f_3 n_3(t)}{2} + \frac{f_4 n_4(t)}{2}\right] + \sigma_1 n_1(t) + \sigma_2 n_2(t) + \sigma_3 n_3(t) + \sigma_4 n_4(t)$$

$$n_2(t) = \sigma_1 n_1(t-1)$$

$$n_3(t) = \sigma_2 n_2(t-1)$$

$$n_4(t) = \sigma_3 n_3(t-1) + \sigma_4 n_4(t-1)$$

$$\frac{\sigma_0}{2} exp[-\beta N(t)] \left[f_2 \sigma_1 n_1(t-1) + f_3 \sigma_2 n_2(t-1) + f_4 (\sigma_3 n_3(t-1) + \sigma_4 n_4(t-1))\right]$$

$$\frac{\sigma_0}{2} exp[-\beta N(t)] (f\sigma)_{min} N(t-1) \leq \sigma_0 exp[-\beta N(t)] \times \left[\frac{f_2 n_2(t)}{2} + \frac{f_3 n_3(t)}{2} + \frac{f_4 n_4(t)}{2}\right] \leq \frac{\sigma_0}{2} exp[-\beta N(t)] (f\sigma)_{max} N(t-1)$$

The Intrinsic Growth Rate

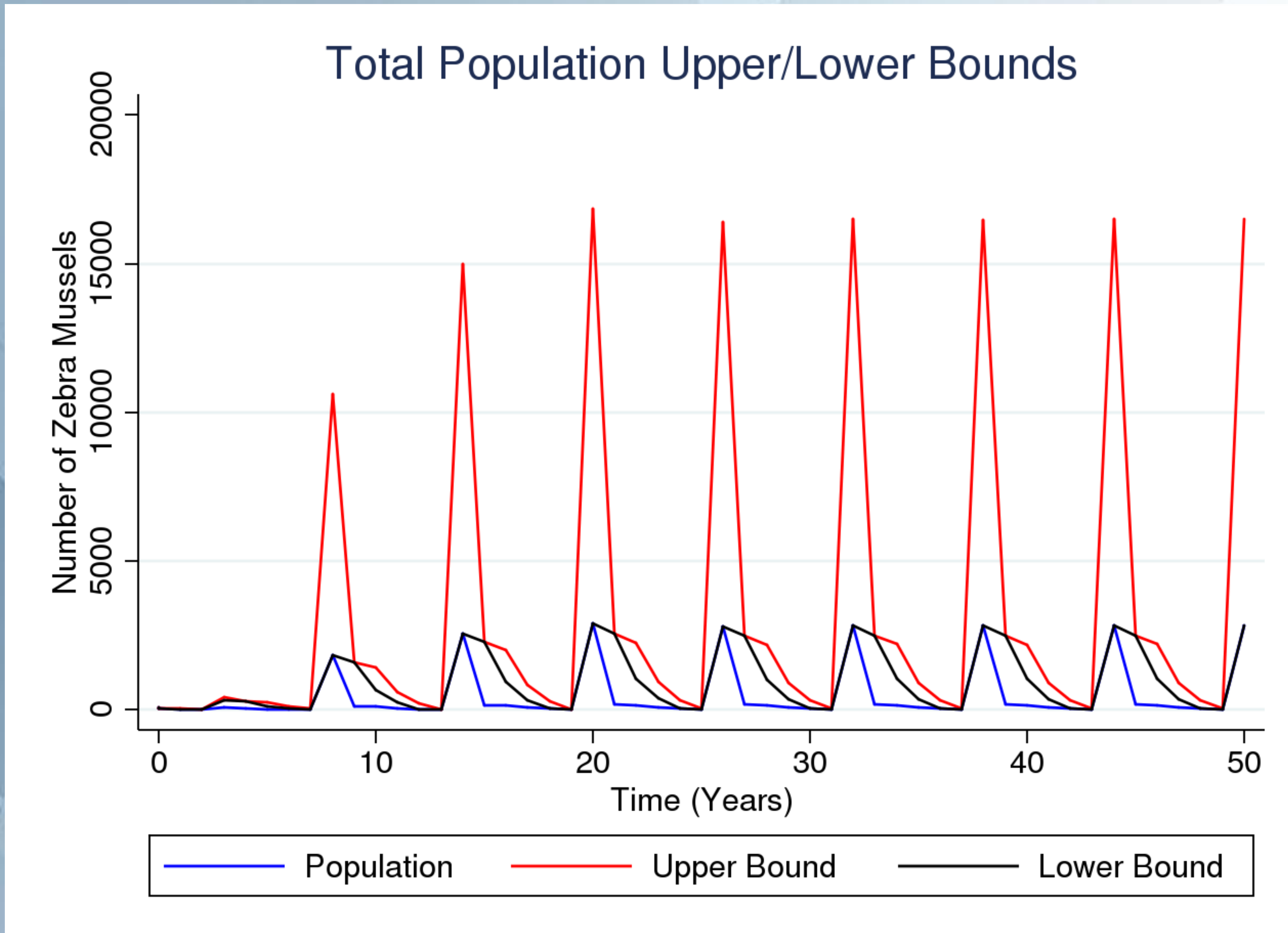
$$\sigma_{min} N(t) + \frac{\sigma_0}{2} exp[-\beta N(t)](f\sigma)_{min} N(t-1)$$

$$\leq N(t+1) \leq$$

$$\sigma_{max} N(t) + \frac{\sigma_0}{2} exp[-\beta N(t)](f\sigma)_{max} N(t-1)$$

$$\sigma_{min} N(t) \leq \sigma_1 n_1(t) + \sigma_2 n_2(t) + \sigma_3 n_3(t) + \sigma_4 n_4(t) \leq \sigma_{max} N(t)$$

The Intrinsic Growth Rate



Stable
Cycle Pattern