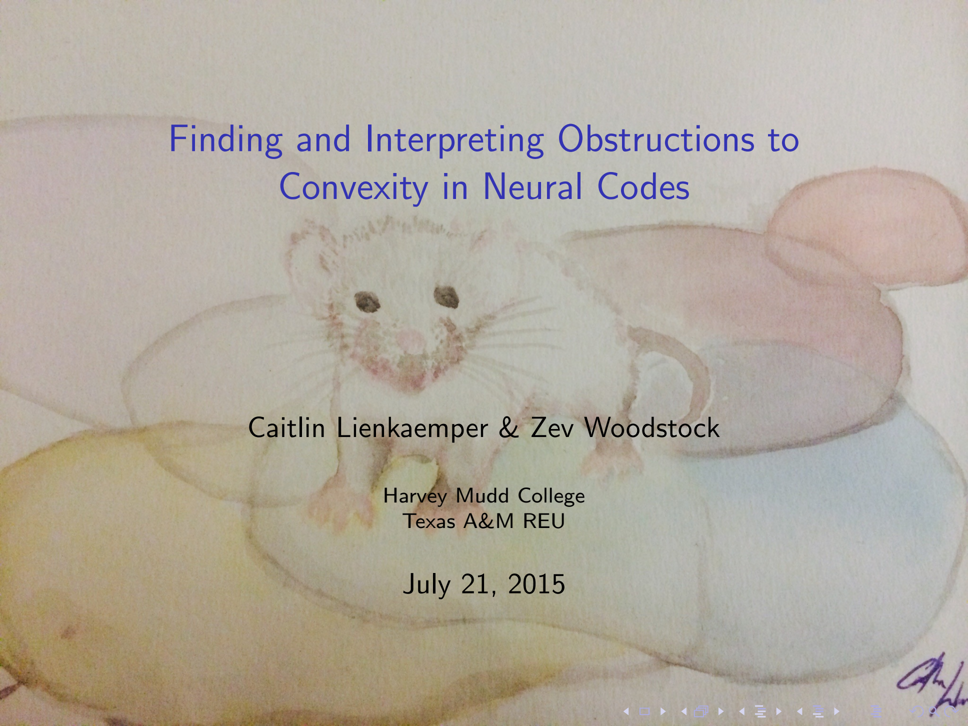


Finding and Interpreting Obstructions to Convexity in Neural Codes

Caitlin Lienkaemper & Zev Woodstock

Harvey Mudd College
Texas A&M REU

July 21, 2015

Neural Codes

How does the brain encode spatial memories?

Definition (neural code)

Given a set of neurons $1, \ldots, n$, a *codeword* is a subset of $1, \ldots, n$ and a *neural code* is a set codewords.

Definition (convex neural code)

A neural code is *convex* if and only if it can describe the activity of a set of place cells with convex receptive fields.

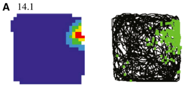


Figure : An experimental example of one place field.

Which neural codes are convex?

A convex neural code

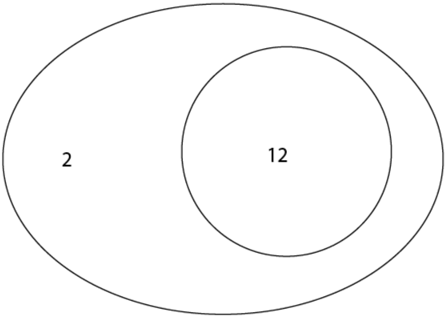


Figure : $\mathcal{C} = \{12, 2\}$

Good Covers

Definition (good cover)

An open cover $\mathcal{U} = U_1, \ldots, U_n$ is a good cover if each U_i is contractible, and if $U_{i_1} \cap \ldots \cap U_{i_m}$ is contractible whenever it is nonempty.

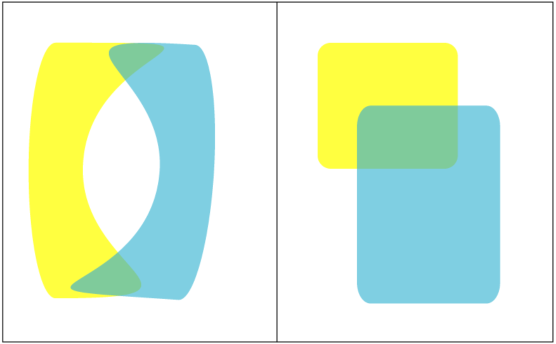


Figure : The code $\mathcal{C} = \{1, 2, 12\}$ is convex. Any convex cover is a good cover.

A way to check for convex codes?

Theorem

If $\mathcal{C}$ is convex, then $\mathcal{C}$ has no local obstructions.

Theorem (stronger)

If $\mathcal{C}$ is comes from a good cover, then $\mathcal{C}$ has no local obstructions.

Conjecture

If neural code $\mathcal{C}$ has no local obstructions, then $\mathcal{C}$ is convex.

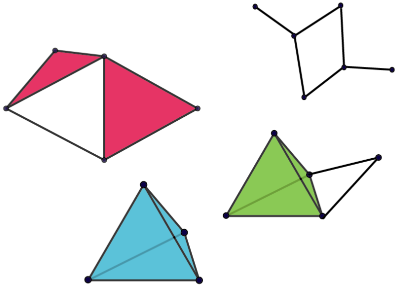
Conjecture (weaker)

If neural code $\mathcal{C}$ has no local obstructions, then $\mathcal{C}$ comes from a good cover.

Simplicial Complexes

Definition (Abstract Simplicial Complex)

An *abstract simplicial complex* is a collection of subsets of a vertex set which is closed under the operation of taking subsets.



Definition (Simplicial Complex of a Neural Code)

The *simplicial complex of a neural code* is the smallest simplicial complex that contains that neural code.

What are local obstructions?

Definition (Maximal Codeword)

Codeword C in neural code $\mathcal{C}$ is maximal if there is no codeword $C' \in C$ such that $C \subset C'$.

What are local obstructions?

Definition (Maximal Codeword)

Codeword C in neural code $\mathcal{C}$ is maximal if there is no codeword $C' \in C$ such that $C \subset C'$.

Definition (Link)

The link of a simplex σ, $Lk(\sigma)$ in a simplicial complex Δ is the set of simplexes $\tau \in \Delta \setminus \sigma$ such that $\sigma \cup \tau \in \Delta$.

What are local obstructions?

Definition (Maximal Codeword)

Codeword C in neural code $\mathcal{C}$ is maximal if there is no codeword $C' \in C$ such that $C \subset C'$.

Definition (Link)

The link of a simplex σ, $Lk(\sigma)$ in a simplicial complex Δ is the set of simplexes $\tau \in \Delta \setminus \sigma$ such that $\sigma \cup \tau \in \Delta$.

Definition (Contractible)

A space is contractible if and only if it is homotopy equivalent to a point.

What are local obstructions?

Definition (Maximal Codeword)

Codeword C in neural code $\mathcal{C}$ is maximal if there is no codeword $C' \in C$ such that $C \subset C'$.

Definition (Link)

The link of a simplex σ, $Lk(\sigma)$ in a simplicial complex Δ is the set of simplexes $\tau \in \Delta \setminus \sigma$ such that $\sigma \cup \tau \in \Delta$.

Definition (Contractible)

A space is contractible if and only if it is homotopy equivalent to a point.

Definition (Local Obstruction)

The neural code $\mathcal{C}$ has a local obstruction if there exists an intersection of maximal codewords $\sigma \notin \mathcal{C}$ such that $Lk(\sigma)$ is not contractible.

Enumerating Simplicial Complexes

Neural codes have only been classfied as convex or not on up to 4 neurons.

We want a classification of neural codes as convex or not, but there are 2147483648 neural codes on only five neurons, so we can't go code by code.

All the action happens on the simplicial complex.

How many simplicial complexes are there on n vertices?
(Sequenced indexed from $n = 1$.)

- Not up to symmetry: 1, 2, 9, 114, 6894, 7785062, 2414627396434, 56130437209370320359966
- Up to symmetry : 1, 2, 5, 20, 180, 16143
- Connected, up to symmetry: 1,1,3,14,157,15940

Classification of Neural Codes on 5 neurons

We can produce a complete description of which neural codes on a simplicial complex have local obstructions by listing the "required faces"–those which are intersections of maximal faces whose links are contractible.

Example:

- Simplicial Complex: $\{\{1,2\},\{2,3\}\}$
- Required Codewords: $\{2\}$.
- Optional Codewords: $\{1\},\{3\}$
- Codes with no local obstructions: $\{\{1,2\},\{2,3\},\{2\}\}$, $\{\{1,2\},\{2,3\},\{2\},\{1\}\}$, $\{\{1,2\},\{2,3\},\{2\},\{3\}\}$, and $\{\{1,2\},\{2,3\},\{2\},\{1\},\{3\}\}$,

We did this for all simplicial complexes on 5 neurons, for a complete description of all neural codes on 5 neurons.

A counterexample:

Let's take a look at the neural code

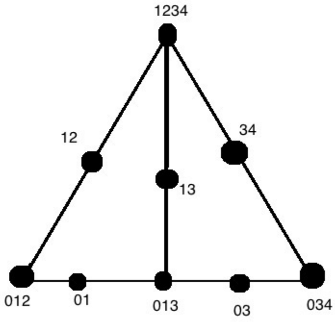
$$\mathcal{C} = \{1234, 012, 034, 013, 12, 34, 13, 01, 03, 3, 1\}.$$

We can verify that it has no local obstructions: the only intersection of maximal codewords missing is 0, and $Lk(0)$ is convex.

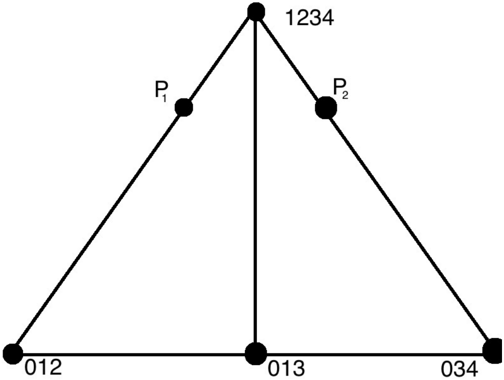
According to our conjecture, $\mathcal{C}$ is convex. So let's find a set of convex receptive fields which generate $\mathcal{C}$.

A forced structure

Recall that if a set A is convex, then if $p \in A$, $q \in A$, then the line segment $\overline{pq} \subset A$, and that the intersection of convex sets must be convex.

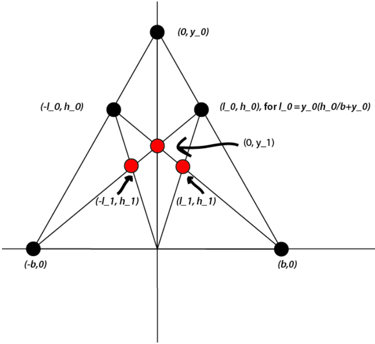


A contradiction



(Animation credit goes to Zev Woodstock.)

We check convergence with a bit of algebra.



Under some assumptions of symmetry, we obtain the sequence

$$h_{n+1} = \frac{bh_n}{2l_0(h_n/h_0) + b}.$$

We prove that this converges to zero.

Where do we go from here?

We've disproved the conjecture that any neural code with no local obstructions is convex. What now?

- Is there a new kind of obstruction we can check for?
  - What would it look like?
- Can we prove the weaker conjecture, that any code with no local obstructions can be realized with a good cover?
  - We at least haven't disproved this yet, and it seems more promising.

Returning to Biology

Reading the mind

- If we are given output from cells know to be place cells, but no location information, we may conclude that place cells are behaving abnormally if neural codes have local obstructions.
- If we are given output from neurons, and we want to check if they might be place cells, we can check if the neural code has local obstructions.

What is the brain doing?

- Like researchers without location data, the brain does not have a record of location independent of that provided by place cells (and a few other cell types).
- Is the brain checking for local obstructions? Probably not.