

On Property P_1 and Spaces of Operators

Stephen Rowe

July 29, 2009

Abstract

A space of operators $S, \subseteq M_n(\mathbb{C})$, is said to have property P_1 if every element of $M_n(\mathbb{C})$ can be written as a rank-1 matrix plus an element of the preannihilator of S . The preannihilator, S_\perp , is the set of all operators, f , such that $Tr(fs) = 0 \quad \forall s \in S$. We investigate the structure of spaces that have property P_1 . We say an algebra A is a maximal P_1 algebra if there does not exist any algebra containing A that also has property P_1 . We show that semi-simple algebras always have property P_1 and that when $A \subset M_n(\mathbb{C})$ is a semi-simple algebra with dimension n , then A is a maximal P_1 algebra.