CONTINUOUS FRAMES ON MANIFOLDS

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Abstract. Continuous moving bases on manifolds play important roles in such areas as differential geometry and mathematical physics. However, not all manifolds have a continuous moving basis for their tangent space. Some important examples of non-parallelizable manifolds are the 2-sphere, Mobius strip, Klein bottle, and projective plane. We show that all of these examples have natural continuous moving Parseval frames of 3 vectors. A Parseval frame can be thought of as a spanning set that can decompose and reconstruct vectors in a similar manner to a basis. Hence, continuous moving Parseval frames can offer a basis-like structure for non-parallelizable manifolds. We also examine continuous dilations of continuous moving Parseval frames. It is an important result in frame theory that every Parseval frame can be dilated to an orthonormal basis. However, due to topological obstructions, a full generalization of this result is impossible for continuous moving Parseval frames. We prove instead that every continuous moving Parseval frame can be continuously dilated to an orthonormal set. Using this result, we show that every manifold with a continuous moving Parseval frame can be embedded in a parallelizable manifold such that the Parseval frame is the projection of a basis for the larger manifold.

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