TOPICS COUSE: COMBINATORICS OF MONOMIAL IDEALS

A cornerstone of combinatorial algebra is the correspondence between finite simplicial complexes and squarefree monomial ideals. This connection made available powerful algebraic techniques to study combinatorial problems, and in fact, the only known way to prove some combinatorial statements about simplicial complexes is using algebra!

One of the most prominent highlights of this theory is the proof (due to Richard Stanley) of the Upper Bound Theorem for simplicial spheres. A simplicial sphere is a simplicial complex homeomorphic to a sphere. If we fix the dimension d and the number of vertices n of such a simplicial complex Δ, the Upper Bound Theorem states that, for $i = 1, \ldots, d$ the number of i-dimensional faces of Δ is less than or equal to the number of i-dimensional faces of a fixed simplicial complex, the face complex of the cyclic polytope with n vertices of dimension d. The key ingredient in Stanley's argument is that the algebraic object attached to a simplicial sphere is a graded Cohen–Macaulay ring, and then the desired combinatorial inequalities precisely correspond to easy Hilbert series inequalities that graded Cohen–Macaulay rings satisfy.

This course aims to present this theory, focusing on combinatorial results with elegant algebraic and topological proofs. The presentation will be self contained.

Topics will include: Simplicial complexes and their face rings, f-vectors and h-vectors, Cohen–Macaulay criteria and the Upper Bound theorem, simplicial polytopes, the g-theorem, shellability, cellular resolutions and Alexander duality.

Proposed Texts:

Combinatorial Commutative Algebra, by Ezra Miller and Bernd Sturmfels.
Combinatorics and Commutative Algebra, by Richard Stanley.

Both of these texts are available for free download at SpringerLink from any campus computer.

Prerequisites: Math 653 or equivalent, or approval of the Instructor.

Learning Outcomes: Students will master topological and algebraic techniques used to solve combinatorial problems.

Instructor Information.

Instructor: Laura Felicia Matusevich
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Grading.

Course Grade: The course grade will be based on regular homework. Collaborative work is allowed, indeed, encouraged, on Homework assignments, but each student must submit their own solution set, written in their own words.

The homework problems will be posted on the course webpage, it is the students' responsibility to regularly check the webpage to obtain the latest homework assignment.

Late Homework and Make Up Exam Policy: Only students with properly documented University-approved excuses may request a make up exam, or an extension on the homework. When possible, such requests should be made in advance. See also *Student Rules: Attendance*, at `http://student-rules.tamu.edu/rule07`.

Other.

Attendance: Attendance of all lectures is required. Make-up work or deadline extensions will be given only in case of absences authorized under *Student Rules: Attendance*, `http://student-rules.tamu.edu/rule07`.

ADA Policy Statement: The Americans with Disabilities Act (ADA) is a federal anti-discrimination statute that provides comprehensive civil rights protection for persons with disabilities. Among other things, this legislation requires that all students with disabilities be guaranteed a learning environment that provides for reasonable accommodation of their disabilities. If you believe you have a disability requiring an accommodation, please contact the Department of Student Life, Services for Students with Disabilities, in Room B118 of the Cain Hall, call 845-1637, or email `disability@tamu.edu`.

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Scholastic Honesty: Always abide by the Aggie Code of Honor: *An Aggie does not lie, cheat, or steal or tolerate those who do.* Please refer to Honor Council Rules and Procedures at `http://aggiehonor.tamu.edu` for more information on academic integrity and scholastic dishonesty.

Contact Information:: Course announcements may occasionally be made via e-mail (e.g. in case of a change to office hours or to clarify problems in homework sets). Students should regularly check their *University-designated* e-mail accounts.