The course will focus on the primary two directions in recent research in harmonic analysis: Bellman function methods and sparse domination. An essential brach of modern harmonic analysis is concerned with weighted inequalities, that is inequalities of the form

$$T : L^p(u) \to L^q(v),$$

where u and v are weights - positive, locally integrable functions $u(x)$, $v(x)$ which determine measures $du := u(x)dx$ and $dv := v(x)$ - and T is some functional operator. Popular instances of T are Calderón-Zygmund operators, commutators, or fractional integral operators.

In 2000, Stefanie Petermichl expressed the Hilbert transform - one of the most foundational operators in harmonic analysis, and the prototype for Calderón-Zygmund operators - as a probabilistic average of dyadic shift operators. These are discrete operators de

ned on a dyadic grid: think first of the standard dyadic grid on $\mathbb{R}$, intervals of the form $[k2^{-j}, (k+1)r^{-j})$ where $k, j, \in \mathbb{Z}$. Petermichl's result has started a whole new branch of harmonic analysis, one in which we can use the simpler, dyadic operators to understand very complicated, continuous operators. For example, an essential geometric property of any dyadic grid on Rn is that if two dyadic cubes intersect, then one must contain the other. This property lies at the base of countless dyadic arguments which prove sharp weighted inequalities for various dyadic operators, which are in turn translated to various continuous operators.

Two amazing dyadic methods have emerged in recent years, both with probabilistic flavors: one coming straight from stochastic differential equations and martingale theory, the Bellman function method, and the other involving dominating operators by certain positive dyadic operators de

ned on a special sparse subset of the dyadic grid, the sparse operator domination method.

The course pre-requisite is only measure theory. The beginning of the course will study some basic objects of harmonic analysis (Maximal function, Hilbert transform, Riesz transforms, Calderón-Zygmund operators, Littlewood-Paley theory, commutators, Muckenhoupt A_p weights) and the rest of the course will be dedicated to studying the recent research directions involving these objects through a dyadic lens.

Prerequisites: none.